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Collection of Materials Presented at the Conference "Geometry and Operator Theory" Dedicated to N. Teleman's 65th Birthday Celebration, Ancona, 20–22 September 2007

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Abstract. This paper forestalls the proceedings of the conference "Geometry and Operator Theory" dedicated to N. Teleman's 65th birthday celebration (Ancona, September 2007)

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1. A SHORT DESCRIPTION OF THE MATHEMATICAL WORK OF NICOLAE TELEMAN

N. Teleman's mathematical formation grew in the famous school of geometry and topology created by Kostake Teleman at the University of Bucharest, which produced many world-known mathematicians. In this environment, N. Teleman became acquainted with the problem of extending the index theory on combinatorial manifolds, which is known as Singer's program (I. M. Singer: Prospects in Mathematics, Princeton Mathematical Studies, No. 70, Princeton, 1970).

N. Teleman graduated from the University of Bucharest in June 1967 with the dissertation "Classification of homogeneous relativistic manifolds" published in St. Cerc. Mat. Tom. 20. No. 1, Bucharest (1968), 67–125, with Prof. Gheorghe Vranceanu as the research supervisor. N. Teleman wrote his PhD thesis in MIT in 1977 with I. M. Singer as the advisor. His work was influenced by researches of D. Sullivan, with whom he collaborated later. In the chronological order, N. Teleman collaborated also with A. Connes, J.-P. Brasselet, A. Legrand, A. Mishchenko, and J. Kubarski. N. Teleman's work influenced researches of numerous mathematicians, including those of A. Connes and S. Donaldson.

Among N. Teleman's main mathematical contributions one should state his extension of the Atiyah–Singer index theorem to the category of Lipschitz manifolds, which belongs to a full body of results due to Riemann-Roch (algebraic category), Hirzebruch (analytic category), Atiyah–Singer (differential category), N. Teleman (Lipschitz category), Donaldson–Sullivan–Teleman–Connes (quasi-conformal category), and Connes (noncommutative geometry).

Characteristic Classes in Differential Setting.

N. Teleman: A geometric definition of some forms of Andre Weil which may be associated with an infinitesimal connection. St. Cerc. Mat. **18** (5), 753–762 (1966).

NOTE: This is the first paper written by N. Teleman while he was still a student. It is shown that, by replacing the parallel transport on an infinitesimal square with the parallel transport on an infinitesimal cube, one obtains an extension of the geometrical definition of the Riemann curvature tensor which produces the Chern character. This work was reconsidered by K. Teleman (K. Teleman: Sur le caractere de Chern d'un fibre vectoriel complexe differentiable. Rev. Roumaine Math. Pures Appl. **12**, 726–731 (1967)) by considering the parallel transport along the sides of an infinitesimal geodesic simplex. This work provides also a geometrical interpretation of the noncommutative

Chern character defined by A. Connes (A. Connes: Noncommutative differential geometry, Inst. Hautes Etudes Sci. Publ. Math. **62**, 257–360 (1985)).

Characteristic Classes in Algebraic Setting.

N. Teleman: A characteristic ring of a Lie algebra extension, I. Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., Ser. 8, **52** (4), 498–506 (1972).

N. Teleman: A characteristic ring of a Lie algebra extension, II. Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., Ser. 8, **52** (5), 708–711 (1972).

NOTE: In this paper is strongly related to the cyclic cohomology theory of characteristic classes of Schatten extensions defined by A. Connes (op. cit.). This paper is also strongly related to the theory of algebroids.

Characteristic Classes in Topological Setting.

N. Teleman: Fibre bundles with involution and characteristic classes. Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., Ser. 8, **54** (1), 49–56 (1973).

N. Teleman: A variant construction of Stiefel–Whitney classes of a topological manifold. Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., Ser. 8, **54** (3), 426–433 (1973).

N. Teleman: Characteristic classes of fibre bundles with involution. Ann. Mat. Pura Appl. (4), **101**, 65–90 (1974).

NOTE: The idea of these papers is to modify the construction of the Steenrod squares by replacing the flipping involution of the factors with an arbitrary involution in a bundle with homologically k-simple fibres to obtain an extension of the Stieffel–Whitney characteristic classes. This leads to an extension of the theory of Stieffel–Whitney classes of real vector bundles to more general homological situations.

N. Teleman: Characteristic classes of S^1-pseudo-actions in fibre bundles, I. Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., Ser. 8, **56** (4), 550–555 (1974).

N. Teleman: Characteristic classes of S^1-pseudo-actions in fibre bundles, II. Chern classes. Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., Ser. 8, **57** (6), 611–618 (1975).

NOTE: The idea of these papers is to sharpen the previous construction to the case of a homological S^1-pseudo-actions in fibre bundles to generalize the Chern classes of complex vector bundles in abstract homological contexts. One possible application would be an elementary construction of the rational Pontryagin classes of topological manifolds.

Characteristic Classes in Classical Global Analysis Setting.

N. Teleman: Global Analysis on Pseudo-Manifolds. PhD thesis, MIT, 1977. Advisor: I. M. Singer.

NOTE: In this paper, foundations for global analysis on pseudo-manifolds are laid. Preliminary results concerning Hodge theory are obtained. These ideas were further adapted to the case of Lipschitz and quasi-conformal manifolds.

N. Teleman: Global Analysis on PL-manifolds. Trans. Amer. Math. Soc. **256** (1), 49–88 (1979).

NOTE: This paper is a slightly modified version of N. Teleman's PhD thesis specified for combinatorial manifolds.

N. Teleman: Combinatorial Hodge Theory and Signature Theorem. In: *Geometry of the Laplace operator* (Proc. Sympos. Pure Math., **36**, Univ. Hawaii, Honolulu, Hawaii, 1979) (Amer. Math. Soc., Providence, R.I., 1980), pp. 287–292.

N. Teleman: Combinatorial Hodge Theory and Signature Operator. Invent. Math. **61** (3), 227–249 (1980).

NOTE: These papers provide an extension of the Hodge theory from the smooth differential setting to the combinatorial manifolds category.

N. Teleman: The Index of Signature Operators on Lipschitz Manifolds. Inst. Hautes Études Sci. Publ. Math., **58**, 39–78 (1983).

NOTE: In this paper N.Teleman shows that the Atiyah–Singer signature operator and its index make sense and continue to hold in the Lipschitz category of vector bundles on Lipschitz manifolds.

D. Sullivan and N. Teleman: An analytic proof of Novikov's theorem on rational Pontrjagin classes. Inst. Hautes Études Sci. Publ. Math., **58**, 79–83 (1983).

NOTE: As an application of the previous paper, N.Teleman obtains an analytical proof of Novikov's theorem on topological invariance of the rational Pontryagin classes on stable manifolds.

N. Teleman: The Index Theorem for Topological Manifolds. Acta Math. **153** (1-2), 117–152 (1984).

NOTE: One obtains an extension of the Atyiah–Singer index theorem from the smooth differential category to the category of topological manifolds.

Characteristic Classes in Distributional Combinatorial Setting.

N. Teleman: The algebra of piecewise differentiable currents on smooth manifolds. J. Funct. Anal., **99** (1), 191–214 (1991).

N. Teleman: Characteristic classes of piecewise differentiable affine connections on smooth manifolds. J. Funct. Anal., **99** (1), 215–222 (1991).

N. Teleman: Local Singular Differential Calculus and Chern Character on Combinatorial Manifolds. IHES/M/72, pp. 1–18, Inst. Hautes Études Sci., Bures-sur-Yvette, Paris, 2000.

NOTE: These papers provide a local construction of the Pontryagin classes in the combinatorial distributional setting.

Characteristic Classes in Noncommutative Geometry Setting.

A. Connes, D. Sullivan, N. Teleman: Quasiconformal mappings, operators on Hilbert space, and local formulae for characteristic classes. Topology **33** (4), 663–681 (1994). Featured review of the AMS.

NOTE: This paper provides an extension of the index theorem in the category of quasi-conformal manifolds.

N. Teleman: Combinatorial Thom–Hirzebruch Characteristic Classes. IHES/M/57, pp. 1–25, Inst. Hautes Études Sci., Bures-sur-Yvette, Paris, 1999.

N. Teleman: Combinatorial Thom–Hirzebruch Characteristic Classes. K-Theory **26** (1), 1–14 (2002).

NOTE: This paper continues the previous paper on quasi-conformal index theory to the particular case of combinatorial manifolds. In this paper N. Teleman constructs a specific signature operator, geometrically and t-locally (t being a small parameter) attached to the combinatorial manifold and one shows that while t goes to zero its index class converges to a well defined homology class which realizes the Thom–Hirzebruch characteristic classes.

N. Teleman: Microlocalization de l'Homologie de Hochschild, C. R. Acad. Sci. Paris Sér. I Math. **326** (11), 1261–1264 (1998).

NOTE: In this paper N.Teleman shows that Connes' theorem on the Hochschild homology of the algebra of smooth functions is the result of three inter-related phenomena: local topological (germ component), differential (infinite jet component) and combinatorial component phenomenon. This paper extends Connes' result from compact manifolds to paracompact manifolds.

N. Teleman: The Classical Limit in Noncommutative Geometry. IHES/M/59, Inst. Hautes Études Sci., Bures-sur-Yvette, Paris, 1999.

NOTE: This paper adapts the results of the previous paper to a better understanding of the cyclic homology.

N. Teleman: Distance Function, Linear Quasi-connections and Chern Character. IHES Prépublications, IHES M/04/27, Inst. Hautes Études Sci., Bures-sur-Yvette, Paris, June 2004.

N. Teleman: Direct Connections and Chern Character. Singularity Theory, Proceedings of the International Conference "Geometry and Topology of Singularities, CIRM-Luminy, May 2005," World Scientific Publishing Company, 2007.

N. Teleman: Direct Connections and Chern Character. Singularity Theory, Proceedings of the International Conference "Geometry and Topology of Singularities, CIRM-Luminy, May 2005," World Scientific Publishing Company, 2007. NOTE: One shows how the Chern character of smooth Riemannian manifolds may be extracted from the geodesic distance function. In the same paper, N. Teleman introduces the notion of quasi-connection, which obtained later the title "direct connection." The notion of direct connection proves to be very relevant in noncommutative geometry.

J. Kubarski, N. Teleman: Linear Direct Connections. Proc. 7th Int. Conf. "The Mathematical Legacy of C. Ehresmann, Betlewo (Polonia), Maggio 2005, pp. 425–436, Banach Center Publica-

tions, Polish Academy of Science, Warszawa 2007

NOTE: In this paper one gives a deeper understanding of the relationship between characteristic classes and direct connections.

N. Teleman: Combinatorial Thom–Hirzebruch Characteristic Classes. K-Theory, Kluwer, Dordrecht, Boston, London, 2002, pp. 1–14.

J.-P. Brasselet, A. Legrand, N. Teleman: Hochschild Homology of Singular Algebras. K-Theory, Kluwer, Dordrecht, Boston, London, 2003, pp. 1–14.

NOTE: This paper provides a study of the Hochschild homology of singular algebras showing interesting new phenomena concerning the deviation of the properties of the de Rham cohomology from the smooth differential setting to the category of singular spaces.

A. S. Mishchenko, N. Teleman: Almost Flat Bundles and Almost Flat Structures. Topological Methods in Non-Linear Analysis, Vol. 26, No. 1, 2005, pp. 75–88. (The volume dedicated to Olga Ladyzhenskaya).

A. S. Mishchenko, N. Teleman: Classifying Space of Almost-Flat Bundles. Trudy Sem. Vektor. Tenzor. Anal. Izd-vo MGU, Moscow, Vol. XXVI, pp. 250–268, 2005 (Russian).

N. Teleman; Modified Hochschild and Periodic Cyclic Homology. Birkhäuser Series "Trends in Mathematics. "C*-algebras and elliptic theory II", pp. 251-265, Basel, 2008.

NOTE: In this paper one proposes a modification of the Hochschild homology introduced with the intent of extending the range of applications of noncommutative geometry from noncommutative differential category to noncommutative topology.

ABOUT THE CONFERENCE "GEOMETRY AND OPERATOR THEORY" DEDICATED TO N. TELEMAN'S 65TH BIRTHDAY CELEBRATION, ANCONA, 20–22 SEPTEMBER, 2007

The conference was held in Ancona (Italy), 20–22 September 2007, and consists of 15 lectures; most of them are presented below. The lectures were concentrated around the topics of mathematical interest of Nicolae Teleman and include some area which have now the title "noncommutative geometry," especially to problems of operator theory related to geometry and topology.

The collection of articles includes 11 papers of participants of the conference. Let us list them.

1) "Noncommutative Geometry and Arithmetics" by Paulo Almeida.

   The paper is a survey on the modern state of interactions between non-commutative geometry and number theory covering principal problems in the subject. The author illustrates how the methods of noncommutative geometry are currently used to tackle problems in class field theory. Noncommutative Geometry allows to think geometrically in situations where the classical notion of space made up with points, is no longer adequate, thus a 'noncommutative space' is needed; a full account of it is given by its main contributor, Alain Connes. The class field theory, i.e., number theory within the realm of Galois theory, is undoubtedly one of the main achievements in arithmetics, leading to an important algebraic machinery. The connection between noncommutative geometry and number theory is one of the many themes treated in many papers, a small part of which the author tries to put in a more down-to-earth perspective, illustrating through an example what should be called an 'application of physics to mathematics,' and our only purpose is to introduce nonspecialists in this beautiful area.
2) "On the Minimal Number of Critical Points of Smooth Mappings between Closed Manifolds" by Dorin Andrica, Louis Funar, and Elena Kudryavtseva.

   The article is devoted to the problem, stated by P. T. Church and J. G. Timourian in 1972, of computing the minimal number of critical points of mappings from one manifold to another one. This invariant is computed here for all surfaces, which completes the problem.
3) "Star Product Geometries" by Paolo Aschieri.

   The paper studies a class of noncommutative geometries obtained from a triangular Drinfeld (in fact, Etingoff–Kazhdan) twist. The general problem addressed in this paper is the construction of infinitesimal symmetries of a noncommutative manifold. In general, for noncommutative manifolds, there is no direct analog of vector fields and geometric structures associated with tangent bundle. However, for the case in which the noncommutative structure is obtained from the commutative one by twisting, like in the Drinfeld's construction of quantum groups associated to the classical compact Lie groups, one can use a procedure

to obtain semiclassical structures, which can be thought of as deformations of the classical differential geometric structures. The author implements this program for the case in which the noncommutative structure is given by applying this procedure in the case of deformed product algebra (*-product) is obtained from a triangular twist.

4) "Dennis Supertrace and the Hochschild Homology of Supermatrices" by Paul A. Blaga.

   The author constructs a generalization of the Dennis trace (for matrices) to the case of the supermatrices over an arbitrary (not necessarily commutative) superalgebra with unit. By analogy with the ungraded case, it is shown how it is possible to use this mapping to construct an isomorphism from the Hochschild homology of the superalgebra to the Hochschild homology of the supermatrix algebra.

5) "Teleman Localization of Hochschild Homology in Singular Setting" by Jean-Paul Brasselet and André Legrand.

   The aim of the paper is to generalize the Hochschild–Kostant–Rosenberg theorem to the case of singular varieties, more precisely, manifolds with boundary and varieties with isolated singularities. In these situations, the authors define suitable algebras of functions and study the localization of the corresponding Hochschild homology. The tool in use is the Teleman localization process. In the case of isolated singularities, the closed Hochschild homology corresponds to the intersection complex that relates the defined objects to intersection homology.

6) "Spinors and Theta Deformations" by Ludwik Dąbrowski.

   The paper clarifies some questions arising in the construction of isospectral deformations and which related to the existence of inequivalent spin structures on tori (used to construct isospectral deformations).

   Namely, the construction of Dirac operators on θ-deformed manifolds is recalled, stressing the aspect of spin structure. The description of multiparameter deformation by A. Connes and M. Dubois-Violette is extended to an arbitrary spin structure.

7) "Algebraic Aspects of the Hirzebruch Signature Operator and Applications to Transitive Lie Algebroids" by Jan Kubarski and Alexander Mishchenko.

   The authors present a general approach to the signature operator, where all known examples are special cases of a single general theorem.

8) "Examples of Gauged Laplacians on Noncommutative Spaces" by Giovanni Landi.

   The paper intends to generalize the construction of gauged Laplace operators to some examples of noncommutative spaces with the physical background of quantum Hall effect. In particular, it is devoted to a so-called toric deformation, with the noncommutative instantons over the theta-deformed S^4.

9) "Buildings Have Finite Asymptotic Dimension" by Jan Dymara and Thomas Schick.

   The authors prove that the asymptotic dimension of any (not necessarily thick or locally finite) building coincides with the asymptotic dimension of an apartment in that building. Along with the theorem of Dranishnikov and Januszkiewicz, which states that apartments in buildings have finite asymptotic dimension, this implies that the building by itself has finite asymptotic dimension.

10) "Construction of Fredholm Representations and a Modification of the Higson–Roe Corona" by A. S. Mishchenko and N. Teleman.

    The paper is devoted to a natural construction of the so-called Fredholm representations of discrete groups, which uses a modification of the Higson–Roe corona. The most interesting case here occurs if the corona is a manifold with boundary, which enables one to construct a sufficiently large family of Fredholm representations.

11) "Cohomological Tautness for Riemannian Foliations" by J. I. Royo Prieto, M. Saralegi-Aranguren, and R. Wolak.

    The paper presents new results on the tautness of Riemannian foliations in their historical context. For a closed manifold, the tautness of a Riemannian foliation can be characterized cohomologically. The authors extend this cohomological characterization to a class of foliations which includes the foliated strata of any singular Riemannian foliation of a closed manifold.