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Poincaré duality and signature for topological manifolds ☆

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Abstract

The signature of the Poincaré duality of compact topological manifolds with local system of coefficients can be described as a natural invariant of nondegenerate symmetric quadratic forms defined on a category of infinite dimensional linear spaces. The objects of this category are linear spaces of the form $W = V \oplus V^*$ where V is abstract linear space with countable base. The space W is considered with minimal natural topology. The symmetric quadratic form on the space W is generated by the Poincaré duality homomorphism on the abstract chain–cochain groups induced by singular simplices on the topological manifold.

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1. Introduction

It is well known that each nondegenerated Hermitian form over complex numbers assigns an integer which is equal to the difference of dimensions of positive and negative subspaces of the form for proper splitting of the underlying space. This integer is called by the signature of Hermitian form and does not depend of reduction to diagonal matrix.

In topology Hermitian forms naturally appear in cohomology groups of orientable manifolds. Given two cohomology classes in the middle dimension of an orientable closed manifold one can take the product of the classes and then take the integral along the fundamental class. In the case of $\dim M = 4k$ we obtain nondegenerated symmetric quadratic form for which the signature is defined.

The signature has natural properties both with respect to disconnected union of manifolds, that is the signature of sum equals to the sum of signatures and with respect to Cartesian product, that is the signature of product equals to the product of signatures. More of that the signature is invariant of oriented bordisms, that is the signature of the boundary of oriented manifold vanishes. All these properties are useful for study of manifolds from the point of view of their classification.

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The natural aim is in building of the signature type invariants for nonsimply connected manifolds with nontrivial coefficient systems. The most natural is to consider a representation of fundamental group of the manifold in a C^*-algebra. Such kind of the signature type invariants belongs to Hermitian K-theory of fundamental group.

But the building of the signature type invariants in Hermitian K-theory faces with an essential obstruction, which is that the cohomology groups for arbitrary local system of coefficients may not be projective modules. This obstruction was overcome in the paper [2] for triangulated combinatorial manifolds. In this case the chain and cochain groups are free finite generated modules. The Poincaré duality can be realized by the intersection operator with fundamental cycle from the cochain module to the chain module and induces the isomorphism in homology groups. Using standard algebraic machinery one can construct a nondegenerated Hermitian form on a free finitely generated module that induces an element in Hermitian K-theory.

So symmetric noncommutative signature can be build for triangulated topological orientable manifolds. Following the argument by M. Gromov [3] (see also [4]) the signature should be constructed for nonsimply connected topological manifolds. If the representation of the fundamental group is finite dimensional then the signature can be calculated without using of the Poincaré complexes of chain groups. In the case of the group algebra of the fundamental group the problem of direct construction of the signature was still open.

The existence of triangulation for arbitrary topological manifolds is still unknown. One can use some cell structure of topological manifolds induced by the handle decomposition which is based on heavy and inefficient theory of microbundles.

All natural constructions, such as Čech homology, singular homology, Alexander–Spanier homology lead to infinitely generated chain and cochain groups.

Two problems appear. The first problem consists in building algebraic category which includes infinitely generated free modules with natural topology and which admits an extension of signature functor to Hermitian K-theory.

The second problem consists in application the signature functor to topological manifolds.

The first step in this direction was done in [5] where the construction was developed for simply connected manifolds. The main idea how to extend the construction of signature to infinitely generated modules over the C^*-algebra $\mathcal{A}$ was presented in the papers [6–8] where all modules were countably generated. We present here general case that includes uncountably generated modules for the chain groups of singular simplices. The main Theorem 5 says that one can construct the noncommutative signature of nonsimply connected manifold staring from the Poincaré complex of singular chains of the manifold.

Here we present a proper category of infinitely generated modules over the C^*-algebra $\mathcal{A}$. Consider direct limit h of the countable sequence (uncountable directed family) of finitely generated free $\mathcal{A}$-modules of increasing dimensions and maps which send basis to basis. The module h is free with countable (uncountable) family of generators. Endow the module h with the topology of direct limit. Let H be the module of all continuous functionals on h, $H \stackrel{\text{def}}{=} \operatorname{Hom}_A(h, A)$. The module H is the inverse limit of free finitely generated modules and has the topology of inverse limit. The modules h and H are mutually dual, that is $\operatorname{Hom}_A(H, A) = h$. In particular for the Čech homology of compact manifold the chain module is inverse limit and the cochain module is direct limit whereas, for singular homology the cochain module is the inverse limit and the chain module is direct (uncountable) limit.

Let

$$\alpha : H \oplus j \to (H \oplus j)^* \equiv h \oplus J$$

be self-adjoint continuous linear bijective map. Here h and j are two direct limits, H and J are inverse limits of A-modules. It turns out, that for α is possible to define the signature type invariant as an element of Hermitian K-theory which coincides with classical signature of Hermitian form in finite dimensional case. Namely, there is a splitting of the module $H \oplus j$ into a direct sum such that the matrix of the operator α has almost hyperbolic form

$$\alpha = \left\{ \begin{matrix} 0 & 0 & \beta \\ 0 & \alpha' & * \\ \beta^* & * & * \end{matrix} \right\},$$

where α' is finite dimensional. By definition the signature of α equals to the signature of α',

$$\operatorname{sign} \alpha \stackrel{\text{def}}{=} \operatorname{sign} \alpha'.$$

This definition is independent of the choice of splitting.

2. Signature and infinite generated modules

Let $\mathcal{A}$ be a separable unital C^*-algebra. Let Λ be an infinite abstract set, $\Lambda^\#$ be the set of all finite subsets in Λ which has natural direction by inclusion. If $\alpha \in \Lambda^\#$ that is $\alpha \subset \Lambda$, then $\mathcal{A}^\alpha$ denotes free finite dimensional $\mathcal{A}$-module with free generators x_i, $i \in \alpha$.

Consider a direction of maps $h^\alpha_{\alpha'} : A^\alpha \to A^{\alpha'}$, $\alpha \subset \alpha'$, induced by natural inclusion of free generators. Then the module h is defined as the direct limit of the spectrum

$$h = \varinjlim\big(A^\alpha, h^\alpha_{\alpha'}\big). \tag{1}$$

The elements of h can be described as families of elements of the algebra $\mathcal{A}$ with finite support, that is $x_i \in \mathcal{A}$, $i \in \Lambda$, $x_i = 0$ for $i \notin \alpha \subset \Lambda$. The $\mathcal{A}$-module h has the natural topology generated by the direct limit.

The family of conjugate modules $(\mathcal{A}^\alpha)^* = \mathrm{Hom}_\mathcal{A}(\mathcal{A}^\alpha, \mathcal{A})$ induces the inverse limit

$$H = \varprojlim\big(\big(A^\alpha\big)^*, \big(h^\alpha_{\alpha'}\big)^*\big), \quad \big(h^\alpha_{\alpha'}\big)^* : \big(A^{\alpha'}\big)^* \to \big(A^\alpha\big)^*. \tag{2}$$

The elements of the module H can be described as arbitrary families of elements of the algebra $\mathcal{A}$: $x_i \in \mathcal{A}$, $i \in \Lambda$. The module H has the natural topology of inverse limit. Two modules, h and H are mutually dual as topological C^*-modules. Namely,

Proposition 1. *One has natural isomorphisms*

$$H = \mathrm{Hom}^{\mathrm{Top}}_\mathcal{A}(h, \mathcal{A}), \tag{3}$$

$$h = \mathrm{Hom}^{\mathrm{Top}}_\mathcal{A}(H, \mathcal{A}). \tag{4}$$

The pairing between H and h has the natural form: $x \in h$, $x = \{x_i,\ i \in \Lambda\}$, $y \in H$, $y = \{y_i,\ i \in \Lambda\}$,

$$\langle x, y\rangle = \sum_{i\in\Lambda} a_i b_i. \tag{5}$$

The sum is defined correctly as one of the vector (x) has finite support.

Consider the submodule ξ of h. We shall call that ξ is the submodule of finite type if there exists a finite subset $\alpha \subset \Lambda$ such that

$$\forall x \in \xi,\ \forall i \notin \alpha, \quad x_i = 0. \tag{6}$$

In general case let $\alpha \subset \Lambda$ be a minimal subset such that

$$\forall x \in \xi,\ \forall i \notin \alpha, \quad x_i = 0. \tag{7}$$

Put $\mathfrak{c} \overset{\mathrm{def}}{=} \#(\alpha)$. Then we shall call that ξ is the submodule of the cardinal type $\mathfrak{c}$. If the submodule ξ has the family of generators of cardinality $\mathfrak{c}$ then ξ is of the cardinal type $\mathfrak{c}$. Inverse, if the projective module ξ is the submodule of the cardinal type $\mathfrak{c}$ then it has at least $\mathfrak{c}$ generators.

Theorem 1. *If the module ξ is not the submodule of finite type, then the dual module ξ^* is not finite or countable generated over the algebra $\mathcal{A}$.*

Theorem 1 is reduced to the case when Λ is countable. The latter was proved in [8].

For arbitrary cardinal number $\mathfrak{c}$ one can prove similar property. Assume that the C^*-algebra $\mathcal{A}$ is separable.

Theorem 2. *If the module ξ is the submodule of the cardinal type $\mathfrak{c}$, then the dual module ξ^* has at least $2^\mathfrak{c}$ generators over the algebra $\mathcal{A}$.*

Lemma 1. *Any linear continuous operator $\varphi \in \mathrm{Hom}(H, h)$ is finite generated that is it is represented as a composition $\varphi = \tilde{\varphi} \circ \mathrm{pr}$,*

$$\begin{array}{ccc} H & \xrightarrow{\mathrm{pr}} & A^n \\ \downarrow{\scriptstyle\varphi} & \swarrow{\scriptstyle\tilde{\varphi}} & \\ h & & \end{array} \tag{8}$$

where pr *is the projection onto coordinates $i \in \alpha \subset \Lambda$, $\#(\alpha) = n$.*

For proof it is sufficient to take the neighborhood of zero in h, which does not contain any linear subspace. For example such a neighbourhood is the set

$$B(0,1) = \{x \in h\colon \forall i \in \Lambda,\ \|x_i\| < 1\}. \tag{9}$$

With respect to the definition of topology on H there is a finite subset $\alpha \subset \Lambda$ such that if $x \in H$ satisfies the condition

$$x_i = 0, \quad \forall i \in \alpha, \tag{10}$$

then $x \in \varphi^{-1}(B(0,1))$. Hence $\varphi(x) = 0$.

Theorem 3. *Let*

$$\alpha : H \oplus j \to (H \oplus j)^* \equiv h \oplus J \tag{11}$$

be linear continuous self-adjoint bijection. Then there is self-dual splitting

$$H \oplus j = H_1 \oplus l \oplus j_1, \qquad (H \oplus j)^* = H_1^* \oplus l^* \oplus j_1^* \tag{12}$$

such that l is a finite generated projective $\mathcal{A}$-module, the operator α has the matrix form

$$\alpha = \left\{ \begin{matrix} 0 & 0 & \beta \\ 0 & \gamma & * \\ \beta^* & * & * \end{matrix} \right\}, \tag{13}$$

and γ, β, β^* are bijections.

Define the signature of α as the equivalence class of the pair (l, γ) in Hermitian K-theory

$$\mathrm{sign}(\alpha) \stackrel{\mathrm{def}}{=} \mathrm{sign}(l, \gamma) \in L_0(\mathcal{A}). \tag{14}$$

Theorem 4. *The class* $\mathrm{sign}(l, \gamma)$ *in $L_0(A)$ does not depend of the choice of the constructions in the previous theorem.*

3. Signature of topological manifolds

Consider topological Hausdorff compact manifold X of dimension n. Let $\pi = \pi_1(X, x_0)$ be fundamental group, let $\mathbf{A} = C[\pi]$ be the group algebra and $\mathcal{A} = C^*[\pi]$ be its completion to C^*-algebra.

Denote be $C_i(X, \mathbf{A})$ the chain group of singular simplices with local coefficient system $\mathbf{A}$ generated by the natural representation of fundamental group $\pi_1(X, x_0)$. Then denote

$$C_i(X, \mathcal{A}) \stackrel{\mathrm{def}}{=} \mathcal{A} \otimes_{\mathbf{A}} C_i(X, \mathbf{A}).$$

All of them are free left $\mathcal{A}$ modules. The cochain groups are defined as

$$C^i(X, \mathcal{A}) \stackrel{\mathrm{def}}{=} \mathrm{Hom}_{\mathcal{A}}\big(C_i(X, \mathcal{A}), \mathcal{A}\big).$$

The manifold X is oriented if there is an element $[X] \in H_n(X, \mathbf{R})$ such that for any point $x_0 \in X$ the restriction of $[X]$ to the homology group $H_n(X, X - x, \mathbf{R}) \approx \mathbf{R}$ is the generator. Then there is the intersection operator (see for example [1])

$$D_{[X]} : C^i(X, \mathcal{A}) \to C_{n-i}(X, \mathcal{A}), \qquad D_{[X]}(\alpha) = [X] \cap \alpha, \tag{15}$$

which induces isomorphism in the homology groups

$$D_{[X]}: H^i(X, \mathcal{A}) \to H_{n-i}(X, \mathcal{A}). \tag{16}$$

Define a formal category of singular algebraic Poincaré complexes following [2]. The difference consists of that all modules are not finite generated. Therefore surgeries of singular algebraic Poincaré complexes differ from that in [2].

Consider a complex of free $\mathcal{A}$-modules $C_* = \{C_i, \delta_i,\ i = 0, \dots, \infty\}$, and dual complex $C^* = \{C^i, \delta^i\}$. We say that we have a singular Poincaré complex of dimension n if the diagram

$$\begin{array}{ccccccccccccccccc}
 & & & & 0 & \longleftarrow & C_0 & \xleftarrow{\delta_1} & C_1 & \xleftarrow{\delta_2} \cdots \xleftarrow{\delta_n} & C_n & \xleftarrow{\delta_{n+1}} & C_{n+1} & \xleftarrow{\delta_{n+2}} & C_{n+2} & \xleftarrow{\delta_{n+3}} & \cdots \\
 & & & & \uparrow & & \uparrow{\scriptstyle D_0} & & \uparrow{\scriptstyle D_1} & & \uparrow{\scriptstyle D_n} & & \uparrow & & & & \\
\cdots & \xleftarrow{\delta^{n+3}} & C^{n+2} & \xleftarrow{\delta^{n+2}} & C^{n+1} & \xleftarrow{\delta^{n+1}} & C^n & \xleftarrow{\delta^n} & C^{n-1} & \xleftarrow{\delta^{n-1}} \cdots \xleftarrow{\delta^1} & C^0 & \longleftarrow & 0 & & & &
\end{array} \tag{17}$$

has the properties:

(1) The image of D_i lies in a finite dimensional coordinate submodule of C_i.
(2) Commutativity and self-adjointness

$$\delta_i D_i = (-1)^i D_{i-1} \delta^{n-i+1}, \tag{18}$$

$$D_i = (-1)^{(n-i)i} D^*_{n-i}. \tag{19}$$

(3) Induced homomorphisms $D_i : H(C^*) \to H(C_*)$ are isomorphisms.

Hence the system $\mathcal{C}(X) = \{C^*(X, \mathcal{A}), \delta, D_{[X]}\}$ is the Poincaré complex of singular chains for the manifold X.

One can similar define a singular Poincaré pair $\{C_*, B_*, \delta, D\}$ of dimension $n + 1$ and the boundary $\{B_*, \delta, E\}$ of dimension n as a singular Poincaré complex where

$$E_{i-1} = \delta_i D_i + (-1)^{i+1} D_{i-1} \delta^{n-i+2}. \tag{20}$$

The notion of singular Poincaré pair generates the bordism relation which is equivalent relation. Two singular Poincaré complexes $c = \{C_*, d_*, D_*\}$ and $c' = \{C'_*, d'_*, D'_*\}$ are called homotopy equivalent if there exists a homomorphism $f : C_* \to C'_*$ such that

$$f_* d_* = d'_* f_*, \qquad D'_* = f D_* f^*, \tag{21}$$

and f induces isomorphism in homology. If two singular Poincaré complexes are homotopy equivalent then they are bordant.

3.1. Infinite dimensional surgeries

Consider the singular Poincaré complex (22) and the left part

$$\begin{array}{ccccccccccc}
 & & & & 0 & \longleftarrow & C_0 & \xleftarrow{\delta_1} & C_1 & \xleftarrow{\delta_2} & \cdots \\
 & & & & \uparrow & & \uparrow{\scriptstyle D_0} & & \uparrow{\scriptstyle D_1} & & \\
\cdots & \xleftarrow{\delta^{n+3}} & C^{n+2} & \xleftarrow{\delta^{n+2}} & C^{n+1} & \xleftarrow{\delta^{n+1}} & C^n & \xleftarrow{\delta^n} & C^{n-1} & \xleftarrow{\delta^{n-1}} & \cdots
\end{array} \tag{22}$$

The image of D_0 lies in a finite dimensional free submodule B, $\operatorname{Im} D_0 \subset B \subset C_0$. Consider a representation of C_0 as direct sum of free modules

$$C_0 = C'_0 \oplus B. \tag{23}$$

One has

$$\begin{array}{ccccccccccc}
 & & & & 0 & \longleftarrow & B \oplus C'_0 & \xleftarrow{\alpha \oplus \beta} & C_1 & \xleftarrow{\delta_2} & \cdots \\
 & & & & \uparrow & & \uparrow{\scriptstyle D_0} & & \uparrow{\scriptstyle D_1} & & \\
\cdots & \xleftarrow{\delta^{n+3}} & C^{n+2} & \xleftarrow{\delta^{n+2}} & C^{n+1} & \xleftarrow{\delta^{n+1}} & C^n & \xleftarrow{\delta^n} & C^{n-1} & \xleftarrow{\delta^{n-1}} & \cdots
\end{array} \tag{24}$$

where β is surjective. Hence one can change the complex (24) to the homotopy equivalent complex

$$\begin{array}{ccccccccccc}
 & & & & 0 & \longleftarrow & B & \xleftarrow{\alpha} & C_1' & \xleftarrow{\delta_2} & \cdots \\
 & & & & \uparrow & & \uparrow{\scriptstyle D_0} & & \uparrow{\scriptstyle D_1} & & \\
\cdots & \xleftarrow{\delta^{n+3}} & C^{n+2} & \xleftarrow{\delta^{n+2}} & C^{n+1} & \xleftarrow{\delta^{n+1}} & C^n & \xleftarrow{\delta^n} & C^{n-1} & \xleftarrow{\delta^{n-1}} & \cdots
\end{array} \tag{25}$$

and using standard finite dimensional surgery to the homotopy equivalent complex

$$\begin{array}{ccccccccccc}
 & & & & 0 & \longleftarrow & 0 & \longleftarrow & C_1'' & \xleftarrow{\delta_2} & \cdots \\
 & & & & \uparrow & & \uparrow{\scriptstyle D_0} & & \uparrow{\scriptstyle D_1} & & \\
\cdots & \xleftarrow{\delta^{n+3}} & C^{n+2} & \xleftarrow{\delta^{n+2}} & C^{n+1} & \xleftarrow{\delta^{n+1}} & C^n & \xleftarrow{\delta^n} & C^{n-1} & \xleftarrow{\delta^{n-1}} & \cdots
\end{array} \tag{26}$$

Finally, for $n = 2k$ one can obtain a homotopy equivalent complex of the form

$$\begin{array}{ccccccccc}
 & & 0 & \longleftarrow & C_k & \xleftarrow{\delta_{k+1}} & C_{k+1} & \xleftarrow{\delta_{k+2}} & \cdots \\
 & & \uparrow & & \uparrow{\scriptstyle D_k} & & \uparrow & & \\
\cdots & \xleftarrow{\delta^{k+2}} & C^{k+1} & \xleftarrow{\delta^{k+1}} & C^k & \longleftarrow & 0 & &
\end{array} \tag{27}$$

and for $n = 2k + 1$ a complex of the form

$$\begin{array}{ccccccccccc}
 & & 0 & \longleftarrow & C_k & \xleftarrow{\delta_{k+1}} & C_{k+1} & \xleftarrow{\delta_{k+2}} & C_{k+2} & \xleftarrow{\delta_{k+3}} & \cdots \\
 & & \uparrow & & \uparrow{\scriptstyle D_k} & & \uparrow{\scriptstyle D_{k+1}} & & \uparrow & & \\
\cdots & \xleftarrow{\delta^{k+3}} & C^{k+2} & \xleftarrow{\delta^{k+2}} & C^{k+1} & \xleftarrow{\delta^{k+1}} & C^k & \xleftarrow{\delta^k} & 0 & &
\end{array} \tag{28}$$

In the case $n = 2k$ let $\varphi : C_k \to \ker \delta^{k+1}$, $\psi : C_k \to C_{k+1}$ such that

$$D_k\varphi + \delta_{k+1}\psi = \mathrm{Id}.$$

The operator $P = \varphi D_k$ is projector with $\mathrm{Im}(\varphi D_k) = \ker \delta^{k+1}$. Really, if $x \in \ker \delta^{k+1}$ then $D_k(\varphi D_k x - x) = (D_k\varphi - \mathrm{Id}) D_k x = \delta_{k+1}\psi x$. This means that in the homology one has $[D_k(\varphi D_k x - x)] = 0$. Since the operator D_k is isomorphic on the level of homology we have that $\varphi D_k x - x = 0$ or $\varphi D_k x = x$.

Hence the diagram (27) is splitted into direct sum of diagrams

$$\begin{array}{ccccccccc}
 & & 0 & \longleftarrow & (\mathrm{Im}\, P^*)^\perp & \xleftarrow{\delta_{k+1}} & C_{k+1} & \xleftarrow{\delta_{k+2}} & \cdots \\
 & & \uparrow{\scriptstyle 0} & & \uparrow{\scriptstyle 0} & & \uparrow{\scriptstyle 0} & & \\
\cdots & \xleftarrow{\delta^{k+2}} & C^{k+1} & \xleftarrow{\delta^{k+1}} & (\mathrm{Im}\, P)^\perp & \longleftarrow & 0 & &
\end{array} \tag{29}$$

and

$$\begin{array}{ccccccccc}
 & & 0 & \longleftarrow & \mathrm{Im}\, P^* & \longleftarrow & 0 & \longleftarrow & \cdots \\
 & & \uparrow{\scriptstyle 0} & & \uparrow{\scriptstyle D_k} & & \uparrow{\scriptstyle 0} & & \\
\cdots & \longleftarrow & 0 & \longleftarrow & \mathrm{Im}\, P & \longleftarrow & 0 & &
\end{array} \tag{30}$$

The last diagram defines the signature as the element of $K_0(\mathcal{A})$.

In the case of $n = 2k + 1$ we should consider the manifold $M \times \mathbf{S}^1$ and the algebra $\mathcal{A}' = \mathbf{C}^*[\pi_1(M \times \mathbf{S}^1)] = \mathbf{C}(\mathbf{S}^1, \mathcal{A})$. Then $K_0(\mathcal{A}') \approx K_0(\mathcal{A}) \oplus K_1(\mathcal{A})$ and the signature of manifold M can be described as the second component of $\mathrm{sign}(M \times \mathbf{S}^1) \in K_0(\mathcal{A}') \approx K_0(\mathcal{A}) \oplus K_1(\mathcal{A})$.

Thus we have the following

Theorem 5. *The Poincaré complex $\mathcal{C}(X)$ of singular chains on the manifold X generates the signature* $\mathrm{sign}(\mathcal{C}(X)) \in K_*(\mathcal{A})$ *that is invariant with respect to both bordisms and homotopy equivalences.*

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