BRITISH-RUSSIAN SEMINAR ON TORIC TOPOLOGY AND HOMOTOPY THEORY

Steklov Mathematical Institute
8 Gubkina str., Moscow, RUSSIA

Third Session: **Tuesday, 4 April 2017, room 530**

10:30–11:20 **Anton Ayzenberg**
Spectral sequences associated with torus actions

11:40–12:30 **Tyrone Cutler**
The homotopy types of $U(n)$-gauge groups over S^4 and $\mathbb{C}P^2$

Lunch: 12:30–14:00

14:00–14:40 **Elizaveta Zhuravleva**
Massey products on moment-angle complexes

15:00–15:50 **Denis Gorodkov**
Combinatorial formulas for Miller–Morita–Mumford classes

This activity is part of the Collaborative Research Programme on Toric Topology between Moscow State University, Steklov Mathematical Institute and University of Southampton, supported by the Royal Society and RFBR.

Organisers: **Victor Buchstaber** (Moscow), **Alexander Gaifullin** (Moscow), **Jelena Grbić** (Southampton), **Taras Panov** (Moscow), **Stephen Theriault** (Southampton)

ABSTRACTS

Anton Ayzenberg

Spectral sequences associated with torus actions

Let M be a $2n$-manifold equipped with a locally standard action of n-dimensional torus T. Such manifolds are classified, at least topologically, by their orbit spaces, characteristic functions and Euler class of the free part of action. The description of the cohomology ring of M in terms of these data is a complicated problem in general. However, the description is known when the topology of the orbit space is simple: if every face of the orbit space is acyclic, then the cohomology ring of M is the quotient of the face ring by a parametric ideal (Masuda–Panov). Spectral sequence approach allows to answer the question in a greater generality. I am going to present some results based on the spectral sequence of the orbit type filtration, and share some thoughts about the Leray spectral sequence of the projection map $M \to M/T$.

Tyrone Cutler

The homotopy types of $U(n)$-gauge groups over S^4 and $\mathbb{C}P^2$

There are many reasons why the fibrewise automorphism group, or gauge group, of a given principal G-bundle is a natural object to study and the topology of these groups can have a profound effect on many physical and geometric invariants associated with the spaces in the G-bundle. It is for this reason the gauge groups of $SU(n)$-bundles over simply connected 4-manifolds have been extensively studied. The case for $G = U(n)$ in the same context, however, has not – the extra low-dimensional topological information in $U(n)$ complicating matters somewhat.

In this talk I present my work studying the homotopy types of $U(n)$-gauge groups over two fundamental 4-manifolds, S^4 and $\mathbb{C}P^2$. I first discuss the case of S^4 as it sets the stage for the more complicated case of $\mathbb{C}P^2$. I give homotopy decompositions for the $U(n)$-gauge groups over S^4 in terms of certain $SU(n)$- and $PU(n)$-gauge groups which are strong enough to allow for a complete enumeration of the distinct homotopy types of the $U(n)$-gauge groups for small n using previous results.

Working now over $\mathbb{C}P^2$ the problem becomes very delicate. I give equivalence statements and p-local decompositions for the $U(n)$-gauge groups, also obtaining homotopy decompositions in certain favourable cases that allow for a complete solution to the enumeration problem. These results are applied in example to the case of $U(2)$ where intuition allows for a clearer understanding of the problem.

Denis Gorodkov

Combinatorial formulas for Miller-Morita-Mumford classes

Miller-Morita-Mumford classes (MMM-classes for short) are characteristic classes for surface bundles (with holes). They were independently introduced by Mumford and Morita in the 1980-s. Often MMM-classes are viewed as elements of integral cohomology of the mapping class group M_g. The present talk is mainly devoted to Kiyoshi Igusa's work "Combinatorial MillerMoritaMumford classes and Witten cycles". We mostly deal with fat graphs (also named ribbon graphs) which are a useful tool for encoding surfaces with holes, as they provide purely combinatorial data. In particular, we review the results from this paper concerning explicit combinatorial formulae for MMM-classes in terms of fat graphs. Such notions as the simplicial nerve of a category and the Stasheff associahedron play an important role in the talk.

Elizaveta Zhuravleva

Massey products on moment-angle complexes

In this talk I will discuss Massey products in the cohomology rings of moment-angle complexes $\mathcal{Z}_{\mathcal{K}}$. In particular, I will prove that there is a non-trivial triple Massey product in the case of $\mathcal{Z}_P = \mathcal{Z}_{\mathcal{K}_P}$, where P is a simple 3-polytope from a large class. For example, it holds if P is a fullerene. The proof is based on the description of the cohomology ring of $\mathcal{Z}_{\mathcal{K}}$ in combinatorial terms and some combinatorial properties of fullerenes.