INTRODUCTION TO TOPOLOGY / TOPOLOGY-1 PROGRAMME

LECTURER: TARAS PANOV

1. Topological spaces, continuous maps, homeomorphisms.
2. Connectivity, compactness, Hausdorffness. A continuous bijective map from a compact space to a Hausdorff space is a homeomorphism.
3. Quotient topology, quotient spaces, examples.
4. Product topology, the universal property.
5. Pushouts and pullbacks (Cartesian and coCartesian squares), the universal properties, examples.
6. Topologies on the function spaces, the compact-open topology, relation to the product topology. The exponential law (without proof).
7. Cylinder, cone, suspension, join.
8. Pointed spaces, their product, wedge and smashed product.
9. Path and loop spaces. The adjunction homeomorphism between maps from a suspension and maps to a loop space.
10. Homotopy, two definitions and their relationship via the exponential law. Homotopy equivalence, contractibility, examples.
11. Cell complexes (CW complexes), axiomatic definition. The operation of attaching a cell. Cell decomposition of a product of cell complexes.
12. Examples of cell complexes: spheres, finite and infinite projective spaces, classical surfaces.
13. Homotopy extension property, connection with retractions.
14. Homotopy extension property for cellular pairs. Quotients by contractible subspaces.
15. Cellular approximation theorem. Homotopical triviality of maps $S^k \to S^n$ with $k < n$.
16. Homotopy of loops. Product of loops, its properties.
17. The fundamental group of a pointed space. Its functorial properties. Connection with homotopy and homotopy equivalences.
18. Fundamental group: change of basepoint.
19. The fundamental group of the circle.
20. Consequences of the calculation of $\pi_1(S^1)$: the nonexistence of a retraction $D^2 \to S^1$, Brouwer's fixed-point theorem, the fundamental theorem of algebra.
21. Free product of groups. Reduced words. Associativity of product of reduced words. A free group. Presentation of a group by generators and relations. Abelianisation.
22. The Seifert–van Kampen theorem.

23. A path-connected cell complex is homotopy equivalent to a cell complex with a single zero-cell.
24. Presentation of the fundamental group of a cell complex by generators and relations.
25. Covering spaces. Examples. Path lifting property.
26. Covering homotopy property. The existence and uniqueness of a covering homotopy for covering spaces.
27. Homomorphism of the fundamental groups induced by a covering map. The cardinality of the preimage of point under a covering map and the index of a subgroup.
28. Lifting properies for maps with respect to coverings (for maps from locally path-connected spaces).
29. The existence and uniqueness of a simply-connected covering of a path-connected, locally path-connected and semilocally simply-connected space. The universal covering.
30. The classification of covering spaces by subgroups in the fundamental group.
31. Graphs. The existence of a maximal tree. The fundamental group of a graph.
32. Coverings of graphs. The Nielsen–Schreier theorem on subgroups of a free group.
33. Locally trivial fibrations. Covering homotopy property.
34. Hurewicz and Serre fibrations.
35. Fibrations and cofibrations. Factorisation theorems.
36. Homotopy groups. Their commutativity.
37. Relative homotopy groups. Homotopy exact sequence of a pair.
38. Homotopy exact sequence of a fibre bundle.
39. The Whitehead theorem.

Recommended literature

[1] Fomenko, A.; Fuchs, D. *Homotopical topology.* Second edition. Graduate Texts in Mathematics, 273. Springer, [Cham], 2016. xi+627 pp.

[2] Hatcher, A. *Algebraic topology.* Cambridge University Press, Cambridge, 2002. xii+544 pp.

[3] Panov, T. E. *Introduction to Topology / Topology-1. Lecture Course.* (In Russian.) `http://higeom.math.msu.su/people/taras/#teaching`

[4] Vassiliev, V. A. *Introduction to topology.* Translated from the 1997 Russian original by A. Sossinski. Student Mathematical Library, 14. American Mathematical Society, Providence, RI, 2001. xiv+149 pp.

[5] Viro, O. Ya.; Ivanov, O. A.; Netsvetaev, N. Yu.; Kharlamov, V. M. *Elementary topology. Problem textbook.* American Mathematical Society, Providence, RI, 2008. xx+400 pp.