

# Dolbeault cohomology of complex manifolds with torus action

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# The moment-angle complex

$\mathcal{K}$  an abstract simplicial complex on the set  $[m] = \{1, 2, \dots, m\}$   
 $I = \{i_1, \dots, i_k\} \in \mathcal{K}$  a **simplex**; always assume  $\emptyset \in \mathcal{K}$ .

Consider the  $m$ -dimensional unit polydisc:

$$\mathbb{D}^m = \{(z_1, \dots, z_m) \in \mathbb{C}^m : |z_i|^2 \leq 1 \text{ for } i = 1, \dots, m\}.$$

The **moment-angle complex** is


$$\mathcal{Z}_{\mathcal{K}} := \bigcup_{I \in \mathcal{K}} \left( \prod_{i \in I} \mathbb{D} \times \prod_{i \notin I} \mathbb{S} \right) \subset \mathbb{D}^m,$$

where  $\mathbb{S}$  is the boundary of the unit disk  $\mathbb{D}$ .


$\mathcal{Z}_{\mathcal{K}}$  has a natural action of the torus  $T^m$ .


When  $\mathcal{K}$  is a simplicial subdivision of sphere (e.g., the boundary of a simplicial polytope),  $\mathcal{Z}_{\mathcal{K}}$  is a topological manifold, called the **moment-angle manifold**.


## Example

1. Let  $\mathcal{K} =$   (the boundary of a triangle). Then

$$\mathcal{Z}_{\mathcal{K}} = (\mathbb{D} \times \mathbb{D} \times \mathbb{S}) \cup (\mathbb{D} \times \mathbb{S} \times \mathbb{D}) \cup (\mathbb{S} \times \mathbb{D} \times \mathbb{D}) = \partial(\mathbb{D}^3) \cong S^5.$$

2. Let  $\mathcal{K} =$   (the boundary of a square). Then  $\mathcal{Z}_{\mathcal{K}} \cong S^3 \times S^3$ .

3. Let  $\mathcal{K} =$   Then  $\mathcal{Z}_{\mathcal{K}} \cong (S^3 \times S^4) \# \cdots \# (S^3 \times S^4)$  (5 times).

4. Let  $\mathcal{K} =$   (three disjoint points). Then

$$\mathcal{Z}_{\mathcal{K}} = (\mathbb{D} \times \mathbb{S} \times \mathbb{S}) \cup (\mathbb{S} \times \mathbb{D} \times \mathbb{S}) \cup (\mathbb{S} \times \mathbb{S} \times \mathbb{D}) \simeq S^3 \vee S^3 \vee S^3 \vee S^4 \vee S^4$$

(not a manifold).

We define an open subset  $U(\mathcal{K}) \subset \mathbb{C}^m$  in a similar way:

$$U(\mathcal{K}) := \bigcup_{I \in \mathcal{K}} \left( \prod_{i \in I} \mathbb{C} \times \prod_{i \notin I} \mathbb{C}^\times \right), \quad \mathbb{C}^\times = \mathbb{C} \setminus \{0\}.$$

$U(\mathcal{K})$  is a toric variety with the corresponding fan given by

$$\Sigma_{\mathcal{K}} = \{\mathbb{R}_{\geq} \langle e_i : i \in I \rangle : I \in \mathcal{K}\},$$

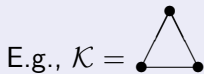
where  $e_i$  denotes the  $i$ -th standard basis vector of  $\mathbb{R}^m$ .

## Theorem

$$(a) \quad U(\mathcal{K}) = \mathbb{C}^m \setminus \bigcup_{\{i_1, \dots, i_k\} \notin \mathcal{K}} \{z_{i_1} = \dots = z_{i_k} = 0\}$$

(the complement of an arrangement of coordinate subspaces);

$$(b) \quad \text{There is a } T^m\text{-equivariant deformation retraction } U(\mathcal{K}) \xrightarrow{\simeq} \mathcal{Z}_{\mathcal{K}}.$$



$$\text{Then } U(\mathcal{K}) = \mathbb{C}^3 \setminus \{z_1 = z_2 = z_3 = 0\} \xrightarrow{\simeq} S^5 = \mathcal{Z}_{\mathcal{K}}.$$

## Simplicial fans and exponential action

Suppose  $\mathcal{K}$  is the underlying complex of a complete simplicial (not necessarily rational) fan  $\Sigma$  in  $W^* \cong \mathbb{R}^n$ .

Then the deformation retraction  $U(\mathcal{K}) \rightarrow \mathcal{Z}_{\mathcal{K}}$  can be realised as the projection onto the orbit space of a smooth free and proper action of a non-compact subgroup  $R \subset (\mathbb{C}^\times)^m$  isomorphic to  $\mathbb{R}^{m-n}$ , as described next.

A simplicial  $\Sigma$  fan is defined by generators  $a_1, \dots, a_m \in W^*$  of its one-dimensional cones and a simplicial complex  $\mathcal{K}$  on  $[m] = \{1, \dots, m\}$ .

For  $I = \{i_1, \dots, i_k\} \in [m]$  we let  $A_I = \{a_i : i \in I\}$ . Then  $\Sigma = \{\text{cone } A_I : I \in \mathcal{K}\}$  provided that the **fan condition** is satisfied:  $\text{relint cone } A_I \cap \text{relint cone } A_J = \emptyset$  for  $I, J \in \mathcal{K}$ ,  $I \neq J$ .

$$\begin{array}{ccc}
0 \longrightarrow V \xrightarrow{\Gamma^*} \mathbb{R}^m \xrightarrow{A} W^* \longrightarrow 0, & A(e_i) = a_i, & V \cong \text{Ker } A \\
\text{Gale duality} & \updownarrow & \\
0 \longrightarrow W \longrightarrow \mathbb{R}^m \xrightarrow{\Gamma} V^* \longrightarrow 0, & \Gamma(e_i) = \gamma_i &
\end{array}$$

Gale dual vector configurations

$A = \{a_1, \dots, a_m\}$  in  $W^* \cong \mathbb{R}^n$  and  $\Gamma = \{\gamma_1, \dots, \gamma_m\}$  in  $V^* \cong \mathbb{R}^{m-n}$ .

Set

$$R = \exp(\text{Ker } A) = \{(e^{\langle \gamma_1, v \rangle}, \dots, e^{\langle \gamma_m, v \rangle}) : v \in V\} \subset (\mathbb{R}^\times)^m \subset (\mathbb{C}^\times)^m$$

Theorem

- (1) The *exponential action* of  $R$  on  $U(\mathcal{K}) \subset \mathbb{C}^m$  is free and proper iff  $\{\mathcal{K}; a_1, \dots, a_m\}$  defines a simplicial fan  $\Sigma = A(\Sigma_{\mathcal{K}})$  in  $W^*$ .
- (2) If  $\Sigma$  is a complete fan ( $|\Sigma| = W^*$ ), then the quotient  $U(\mathcal{K})/R$  is  $T^m$ -equivariantly homeomorphic to the moment-angle manifold  $\mathcal{Z}_{\mathcal{K}}$ .

## Holomorphic exponential actions

$\tilde{V} = (V, \mathcal{J}) \cong \mathbb{C}^\ell$ , where  $\mathcal{J}: V \rightarrow V$  is a complex structure on  $V \cong \mathbb{R}^{m-n}$  (assuming  $m - n = 2\ell$  is even).

$\Gamma = \{\gamma_1, \dots, \gamma_m\}$  a vector configuration in  $\tilde{V}^* = \text{Hom}_{\mathbb{C}}(\tilde{V}, \mathbb{C})$ .  
Complex bilinear pairing  $\tilde{V}^* \times \tilde{V} \rightarrow \mathbb{C}$

$$\langle \gamma, v \rangle_{\mathbb{C}} = \langle \gamma, v \rangle_{\mathbb{R}} - i \langle \gamma, \mathcal{J}v \rangle_{\mathbb{R}}.$$

The complex-analytic subgroup

$$H = \{(e^{\langle \gamma_1, v \rangle_{\mathbb{C}}}, \dots, e^{\langle \gamma_m, v \rangle_{\mathbb{C}}}) : v \in \tilde{V}\} \subset (\mathbb{C}^\times)^m$$

acts holomorphically on

$$U(\mathcal{K}) = (\mathbb{C}, \mathbb{C}^\times)^{\mathcal{K}} = \mathbb{C}^m \setminus \bigcup_{\{i_1, \dots, i_p\} \notin \mathcal{K}} \{\mathbf{z} : z_{i_1} = \dots = z_{i_p} = 0\}$$

## Theorem

- (1) The *holomorphic exponential action* of  $H \cong \mathbb{C}^\ell$  on  $U(\mathcal{K}) \subset \mathbb{C}^m$  is free and proper iff  $\{\mathcal{K}; a_1, \dots, a_m\}$  defines a simplicial fan  $\Sigma = A(\Sigma_{\mathcal{K}})$  in  $W^*$ .
- (2) If  $\Sigma$  is a complete fan ( $|\Sigma| = W^*$ ), then the quotient  $U(\mathcal{K})/H$  is  $T^m$ -equivariantly homeomorphic to the moment-angle manifold  $\mathcal{Z}_{\mathcal{K}}$ .

## Corollary

Let  $\mathcal{K}$  be the underlying complex of a complete simplicial fan (a *star-shaped sphere triangulation*) and  $m - n = 2\ell$  is even.

Then the moment-angle complex  $\mathcal{Z}_{\mathcal{K}} = (D^2, S^1)^{\mathcal{K}}$  has a structure of a compact complex manifold.

## Example (holomorphic torus)

Exponential action of  $H \cong \mathbb{C}$  on  $\mathbb{C}^2$  defined by a configuration  $\{\gamma_1, \gamma_2\}$ :

$$H = \{(e^{\gamma_1 v} z_1, e^{\gamma_2 v} z_2) : v \in \mathbb{C}\}, \quad (z_1, z_2) \mapsto (e^{\gamma_1 v} z_1, e^{\gamma_2 v} z_2).$$

Let  $\mathcal{K} = \{\emptyset\}$  on  $[2] = \{1, 2\}$ , so that  $U_{\mathbb{C}}(\mathcal{K}) = (\mathbb{C}^\times)^2$ . If  $\mathbb{R}\langle\gamma_1, \gamma_2\rangle = \mathbb{R}^2$ , then the restriction of the action to  $(\mathbb{C}^\times)^2$  is free and proper. The quotient

$$(\mathbb{C}^\times)^2 / \tilde{V} \cong \mathbb{C} / (\mathbb{Z}\langle\gamma_1, \gamma_2\rangle) \cong T^2$$

is a one-dimensional complex torus.

More generally, let  $\Gamma = \{\gamma_1, \dots, \gamma_m\}$  be a real basis in  $\tilde{V} \cong \mathbb{C}^\ell$ . Then  $\mathcal{A}$  is a configuration of  $m$  zero vectors in  $W^* = \{0\}$ ,  $\mathcal{K} = \{\emptyset\}$  on  $[m] = [2\ell]$  and  $U_{\mathbb{C}}(\mathcal{K}) = (\mathbb{C}^\times)^m$ . The quotient  $(\mathbb{C}^\times)^m / \tilde{V} \cong \tilde{V} / \Lambda \cong T^{2\ell}$  is a holomorphic torus. Any holomorphic torus can be obtained in this way.

## Proposition

*A complex moment-angle manifold  $\mathcal{Z}_{\mathcal{K}} = U_{\mathbb{C}}(\mathcal{K}) / \tilde{V}$  is non-Kähler, unless it is a holomorphic torus.*

## Irrational toric varieties

If the space  $V$  is **rational** with respect to  $\Gamma = \{\gamma_1, \dots, \gamma_m\}$ , then

$$L = \{v \in V : \langle \gamma_k, v \rangle \in \mathbb{Z} \text{ for } k = 1, \dots, m\}.$$

is a full rank lattice in  $V$ , and  $L^* = \mathbb{Z}\langle \gamma_1, \dots, \gamma_m \rangle \subset V^*$ .

The Gale dual rational vector configuration  $A = \{a_1, \dots, a_m\}$  spans a full rank lattice  $N = M^*$  in  $W^*$ .

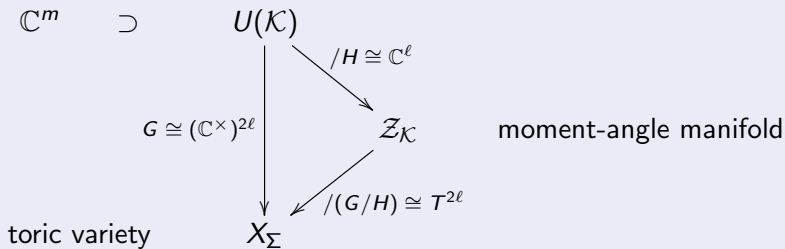
The algebraic torus  $\mathbb{C}_L^\times = L \otimes_{\mathbb{Z}} \mathbb{C}^\times = V_{\mathbb{C}} / (2\pi i L)$  embeds in  $(\mathbb{C}^\times)^m$  as the closed algebraic subgroup

$$\begin{aligned} G &= \exp \Gamma_{\mathbb{C}}^*(V_{\mathbb{C}}) = \exp(\text{Ker } A_{\mathbb{C}}: \mathbb{C}^m \rightarrow W_{\mathbb{C}}^*) \\ &= \left\{ (t_1, \dots, t_m) \in (\mathbb{C}^\times)^m : \prod_{i=1}^m t_i^{\langle a_i, w \rangle} = 1, \quad w \in M \right\}. \end{aligned}$$

The fan  $\Sigma = A(\Sigma_{\mathcal{K}})$  is rational and defines a **toric variety**

$$X_{\Sigma} = U(\mathcal{K})/G = \mathcal{Z}_{\mathcal{K}}/F.$$

Rational case:



A perturbation of the vector configuration  $\mathbf{A}$  destroys the rationality of the fan  $\Sigma = \{\mathcal{K}, \mathbf{A}\}$ , the subgroup  $G \subset (\mathbb{C}^\times)^m$  ceases to be closed, the closed holomorphic tori in the fibres of the bundle  $\mathcal{Z}_{\mathcal{K}} \rightarrow X_\Sigma$  “open up”, and the fibre bundle turns into a **holomorphic foliation**  $\mathcal{F}$  of  $\mathcal{Z}_{\mathcal{K}}$  with noncompact leaves  $F = G/H$ .

The holomorphic foliated manifolds  $(\mathcal{Z}_{\mathcal{K}}, \mathcal{F})$  model **irrational deformations** of toric varieties.

# De Rham and Dolbeault cohomology (rational case)

The **face ring** (the **Stanley–Reisner ring**) of  $\mathcal{K}$  is

$$\mathbb{C}[\mathcal{K}] := \mathbb{C}[v_1, \dots, v_m] / I_{\mathcal{K}} = \mathbb{C}[v_1, \dots, v_m] / (v_{i_1} \cdots v_{i_k} : \{i_1, \dots, i_k\} \notin \mathcal{K}),$$

where  $\mathbb{C}[v_1, \dots, v_m]$  is the polynomial algebra,  $\deg v_i = 2$ , and  $I_{\mathcal{K}}$  is the **Stanley–Reisner ideal**.

## Proposition

*The  $T^m$ -equivariant cohomology is given by*

$$H_{T^m}^*(\mathcal{Z}_{\mathcal{K}}) = H_{T^m}^*(U(\mathcal{K})) \cong \mathbb{C}[\mathcal{K}].$$

The toric variety  $X_\Sigma$  is Kähler (equivalently, projective) if and only if  $\Sigma$  is the normal fan of a nonsingular (Delzant) polytope  $P$ .

### Theorem (Danilov)

*The Dolbeault cohomology of complete nonsingular  $X_\Sigma$  is given by*

$$H_{\bar{\partial}}^{*,*}(X_\Sigma) \cong \mathbb{C}[v_1, \dots, v_m]/(I_\Sigma + J_\Sigma),$$

where  $v_i \in H_{\bar{\partial}}^{1,1}(X_\Sigma)$ ,  $I_\Sigma$  is the Stanley–Reisner ideal,  $J_\Sigma$  is the ideal generated by the linear forms  $\sum_{k=1}^m \langle a_k, w \rangle v_k$ ,  $a_k = A(e_k)$  are the generators of 1-dim cones of  $\Sigma$ ,  $w \in W$ .

The nonzero Hodge numbers are given by  $h^{p,p}(X_\Sigma) = h_p$ , where  $h(\Sigma) = (h_0, h_1, \dots, h_n)$  is the ***h-vector*** of  $\Sigma$ .

## Theorem (Buchstaber-P.)

The de Rham cohomology ring of  $\mathcal{Z}_{\mathcal{K}}$  is given by

$$\begin{aligned} H^*(\mathcal{Z}_{\mathcal{K}}) &\cong \operatorname{Tor}_{\mathbb{C}[v_1, \dots, v_m]}(\mathbb{C}[\mathcal{K}], \mathbb{C}) \\ &\cong H(\Lambda[u_1, \dots, u_m] \otimes \mathbb{C}[\mathcal{K}], d) \quad du_i = v_i, \quad dv_i = 0 \\ &\cong H(\Lambda[t_1, \dots, t_{m-n}] \otimes H^*(X_{\Sigma}), d) \quad \Lambda[t_1, \dots, t_{m-n}] = H^*(T_L) \\ &\cong \bigoplus_{l \in [m]} \tilde{H}^{*-|l|-1}(\mathcal{K}_l). \end{aligned}$$

Here  $H(\Lambda[t_1, \dots, t_{m-n}] \otimes H^*(X_{\Sigma}), d)$  is the  $E_3$  term of the Serre spectral sequence of the principal bundle  $\mathcal{Z}_{\mathcal{K}} \rightarrow X_{\Sigma}$  with fibre  $T_L \cong T^{m-n}$ .

## Theorem (P.-Ustinovsky)

Let  $\Sigma$  be a rational fan,  $\mathcal{Z}_{\mathcal{K}} \rightarrow X_{\Sigma}$  a holomorphic principal bundle with fibre  $\tilde{T}_L = \tilde{V}^*/L^* \cong T^{2\ell}$ .

Then the Dolbeault cohomology ring of  $\mathcal{Z}_{\mathcal{K}}$  is given by

$$H_{\bar{\partial}}^{*,*}(\mathcal{Z}_{\mathcal{K}}) \cong H(H_{\bar{\partial}}^{*,*}(\tilde{T}_L) \otimes H_{\bar{\partial}}^{*,*}(X_{\Sigma}), d),$$

where

$$\begin{aligned} H_{\bar{\partial}}^{*,*}(\tilde{T}_L) &= \Lambda(\tilde{V} \oplus \overline{\tilde{V}}) = \Lambda[\xi_1, \dots, \xi_{\ell}, \eta_1, \dots, \eta_{\ell}] \\ H_{\bar{\partial}}^{*,*}(X_{\Sigma}) &\cong \mathbb{C}[v_1, \dots, v_m]/(I_{\mathcal{K}} + J_{\Sigma}), \end{aligned}$$

$\xi_j \in H_{\bar{\partial}}^{1,0}(\tilde{T}_L) = \tilde{V}$ ,  $\eta_j \in H_{\bar{\partial}}^{0,1}(\tilde{T}_L) = \overline{\tilde{V}}$ ,  $dv_j = d\eta_j = 0$ ,  $d\xi_j = c(\xi_j)$ ,  
 $c: H_{\bar{\partial}}^{1,0}(\tilde{T}_L) \rightarrow H_{\bar{\partial}}^{1,1}(X_{\Sigma})$  is the holomorphic first Chern class map.

## Corollary

- (a) *The Borel spectral sequence of the holomorphic fibration  $\mathcal{Z}_{\mathcal{K}} \xrightarrow{\tilde{T}_L} X_{\Sigma}$  (converging to Dolbeault cohomology of  $\mathcal{Z}_{\mathcal{K}}$ ) collapses at the  $E_3$  page;*
- (b) *The Frölicher spectral sequence (with  $E_1 = H_{\bar{\partial}}^{*,*}(\mathcal{Z}_{\mathcal{K}})$ , converging to  $H^*(\mathcal{Z}_{\mathcal{K}})$ ) collapses at  $E_2$ .*

## Basic cohomology

$M$  a manifold with an action of a connected Lie group  $G$ ,  $\mathfrak{g} = \text{Lie } G$ .

$$\Omega(M)_{\text{bas}, G} = \{\omega \in \Omega(M) : \iota_{\xi}\omega = L_{\xi}\omega = 0 \text{ for any } \xi \in \mathfrak{g}\},$$

$H_{\text{bas}, G}^*(M) = H(\Omega(M)_{\text{bas}, G}, d)$  the **basic cohomology** of  $M$ .

$S(\mathfrak{g}^*)$  the symmetric algebra on  $\mathfrak{g}^*$  with generators of degree 2.

The **Cartan model** is

$$\mathcal{C}_{\mathfrak{g}}(\Omega(M)) = ((S(\mathfrak{g}^*) \otimes \Omega(M))^{\mathfrak{g}}, d_{\mathfrak{g}}),$$

where  $(S(\mathfrak{g}^*) \otimes \Omega(M))^{\mathfrak{g}}$  denotes the  $\mathfrak{g}$ -invariant subalgebra.

An element  $\omega \in \mathcal{C}_{\mathfrak{g}}(\Omega(M))$  is a “ $\mathfrak{g}$ -equivariant polynomial map from  $\mathfrak{g}$  to  $\Omega(M)$ ”. The differential  $d_{\mathfrak{g}}$  is given by

$$d_{\mathfrak{g}}(\omega)(\xi) = d(\omega(\xi)) - \iota_{\xi}(\omega(\xi)).$$

## Theorem

$$H_{\text{bas}, G}^*(M) \cong H(\mathcal{C}_{\mathfrak{g}}(\Omega(M)), d_{\mathfrak{g}}).$$

If in addition  $G$  is a compact, then

$$H_{\text{bas}, G}^*(M) \cong H_G^*(M) = H^*(EG \times_G M) \quad \text{the equivariant cohomology.}$$

**Basic de Rham and Dolbeault cohomology** of the foliated manifold  $(\mathcal{Z}_{\mathcal{K}}, \mathcal{F})$  (irrational toric variety) is like the cohomology of the toric quotient  $X_{\Sigma} = \mathcal{Z}_{\mathcal{K}} / \widetilde{T}_L$  in the rational case:

### Theorem (Ishida–Krutowski–P)

*There is an isomorphism of algebras:*

$$H_{\text{bas}, \bar{\partial}}^{*,*}(\mathcal{Z}_{\mathcal{K}}, \mathcal{F}) \cong \mathbb{C}[v_1, \dots, v_m] / (I_{\mathcal{K}} + J_{\Sigma}),$$

where  $v_i \in H_{\text{bas}, \bar{\partial}}^{1,1}(\mathcal{Z}_{\mathcal{K}}, \mathcal{F})$ ,  $i = 1, \dots, m$ ,

$I_{\mathcal{K}}$  is the Stanley–Reisner ideal, generated by

$$v_{i_1} \cdots v_{i_k} \quad \text{with } \{i_1, \dots, i_k\} \notin \mathcal{K},$$

and  $J_{\Sigma}$  is the ideal generated by the linear forms

$$\sum_{i=1}^m \langle a_i, w \rangle v_i, \quad w \in W.$$

The proof of the theorem is based on the following formality result. Let  $\mathfrak{t} = \text{Lie}(T^m) \cong \mathbb{R}^m$  and consider the Cartan model

$$\mathcal{C}_t(\Omega(\mathcal{Z}_{\mathcal{K}})) = ((S(\mathfrak{t}^*) \otimes \Omega(\mathcal{Z}_{\mathcal{K}}))^{T^m}, d_t).$$

Since  $T^m$  is compact, we get

$$H(\mathcal{C}_t(\Omega(\mathcal{Z}_{\mathcal{K}}))) = H_{T^m}^*(\mathcal{Z}_{\mathcal{K}}) = \mathbb{C}[v_1, \dots, v_m]/I_{\mathcal{K}}.$$

## Lemma

*The DGA  $\mathcal{C}_t(\Omega(\mathcal{Z}_{\mathcal{K}}))$  is formal. Furthermore, there is a zigzag of quasi-isomorphisms of DGAs between  $\mathcal{C}_t(\Omega(\mathcal{Z}_{\mathcal{K}}))$  and  $H_{T^m}(\mathcal{Z}_{\mathcal{K}})$  which respect the  $S(\mathfrak{t}^*)$ -module structure.*

# Dolbeault cohomology in the irrational case

## Theorem (P–Ustinovsky, Krutowski–P)

*The Dolbeault cohomology algebra of  $\mathcal{Z}_{\mathcal{K}}$  is given by*

$$H_{\bar{\partial}}^{*,*}(\mathcal{Z}_{\mathcal{K}}) \cong H(\Lambda(\tilde{V} \oplus \overline{\tilde{V}}) \otimes H_{\text{bas}, \bar{\partial}}^{*,*}(\mathcal{Z}_{\mathcal{K}}, \mathcal{F}), d),$$

*where  $d(\overline{\tilde{V}}) = 0$  and  $d: \tilde{V} \rightarrow H_{\text{bas}, \bar{\partial}}^{1,1}(\mathcal{Z}_{\mathcal{K}}, \mathcal{F})$  is the “foliated Chern class”.*

## References

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