COBORDISMS AND FINITE GROUP ACTIONS IN THE WORK OF TARAS PANOV

In works of T. Panov [5]–[7] the cobordism theory and formal group laws are applied to the study of the fixed point sets of $\mathbb{Z}/p$-actions.

Characteristic classes and cobordisms of manifolds with simple $\mathbb{Z}/p$-action. A $\mathbb{Z}/p$-action on a manifold M is called *simple* if all connected components of the fixed point set have trivial normal bundle. If additionally the tangent representations at the fixed points are same for all fixed submanifolds of the same dimension, then the action is *strictly simple*. In 1964 Conner and Floyd [2] proved that a cobordism class contains a manifold with a strictly simple $\mathbb{Z}/p$-action if and only if all its characteristic numbers are zero mod p. S. P. Novikov in his pioneering work [3] of 1967 brought the methods of formal group laws into cobordism theory. This lead to a new approach to cobordism classification problems related to $\mathbb{Z}/p$-actions. In particular, a significant breakthrough in the problem of describing the cobordism ring ideal generated by manifolds with a simple $\mathbb{Z}/p$-action was achieved in [1]. However, the problem itself had remained unsolved until 1998, when T. Panov obtained its effective solution both in terms of the coefficients of the universal *formal group law of geometric cobordisms* and in terms of divisibility of characteristic numbers.

Hirzebruch genera of manifolds with $\mathbb{Z}/p$-action. The theme of cobordisms of $\mathbb{Z}/p$-actions was further developed in works of Panov [5], [7], where he obtained formulae calculating most important *Hirzebruch genera* of manifolds with simple $\mathbb{Z}/p$-action in terms of the invariants of the action. Hirzebruch genera, such as the Todd genus, signature or elliptic genus, constitute an important family of multiplicative cobordism invariants of manifolds. The problem of calculating these genera for manifolds with $\mathbb{Z}/p$-action was posed in Buchstaber and Novikov's work [1], where the answer was obtained for the Todd genus. In Panov's work a general formula was obtained, expressing an arbitrary genus in terms of the weights of the tangent representations at the fixed points and the cobordisms classes of the fixed submanifolds. Most important particular cases were detailedly treated. These include; the signature, the χ_y-genus, the $\hat{A}$-genus, the elliptic genus, and some other genera arising as the indices of complexes of elliptic operators. In his calculations, T. Panov used two different approaches; one due to Buchstaber–Novikov, which reduces the calculation of a genus to calculating a certain number-theoretical trace of field extension, and another based on the application of the Atiyah–Singer index theorem. The comparison of the formulae obtained by these two different methods led to non-trivial identities for Legendre and Chebyshev polynomials. The latter arise as the coefficients of the logarithm of the corresponding formal group laws.

The results of these works of T. Panov constitute his PhD thesis, finished in 1999. The methods and ideas have since been implemented in the subsequent work of Panov on *toric topology*.

References

[1] В. М. Бухштабер, С. П. Новиков. *Формальные группы, степенные системы и операторы Адамса*, Матем. сборник **84 (126)** (1971), стр. 81–118 (in Russian). English translation: Victor M. Bukhshtaber and Sergei P. Novikov. *Formal groups, power systems and Adams operators.* Math. USSR, Sbornik **13** (1971), 80–116.

[2] P. E. Conner and E. E. Floyd. *Differentiable periodic maps.* Springer-Verlag, Berlin, 1964.

[3] С. П. Новиков. *Методы алгебраической топологии с точки зрения теории кобордизмов.* Изв. АН СССР, серия матем. **31** (1967), стр. 855-951 (in Russian). English translation: Sergei P. Novikov. *The methods of algebraic topology from the viewpoint of cobordism theory.* Math. USSR, Izvestiya **1** (1967), 827–913.

[4] С. П. Новиков. *Операторы Адамса и неподвижные точки.* Изв. АН СССР, серия матем. **32** (1968), стр. 1245–1263 (in Russian). English translation: Sergei P. Novikov. *Adams operators and fixed points.* Math. USSR, Izvestiya **2** (1968), 1193–1211.

[5] Т. Е. Панов. *Эллиптический род для многообразий с действием группы* $\mathbb{Z}/p$. Успехи мат. наук **52** (1997), вып. 2 (1997), стр. 181–182 (in Russian). English translation: Taras E. Panov. *Elliptic genus for manifolds with an action of the group* $\mathbb{Z}/p$. Russian Math. Surveys **52** (1997), no. 2, 418–419.

[6] Т. Е. Панов. *Классификация с точностью до кобордизма многообразий, несущих простое действие группы* $\mathbb{Z}/p$. Мат. заметки **63** (1998), вып. 2, стр. 260–268 (in Russian). English translation: T. E. Panov. *Classification up to cobordism of manifolds that carry a simple action of the group* $\mathbb{Z}/p$. Math. Notes **63** (1998), no. 1–2, 225–232; arXiv:math.AT/9908166.

[7] Т. Е. Панов. *Вычисление родов Хирцебруха многообразий, несущих действие группы* $\mathbb{Z}/p$ *через инварианты действия.* Известия РАН, сер. матем. **62** (1998), вып. 3, стр. 87–120 (in Russian). English translation: T. E. Panov. *Calculation of Hirzebruch genera of manifolds with* $\mathbb{Z}/p$ *group action in terms of the invariants of the action.* Izvestiya: Mathematics **62** (1998), no. 3, 515–548; arXiv:math.AT/9909081.