4. Result

Γ : 2n-valent hypertorus graph

$$\begin{cases} \text{(i)} \ \forall L \subset \Gamma : (2n-2)\text{-valent subgr} \\ \quad \exists! H, \overline{H} : \text{hyper facet} \\ \text{(ii)} \ L_1, \cdots, L_i \subset \Gamma : (2n-2)\text{-valent subgr} \\ \quad \Rightarrow L_1 \cap \cdots \cap L_i = \phi \text{ or connected} \end{cases}$$

$$\Rightarrow \boxed{H_T^*(\Gamma, \alpha) \cong \mathbb{Z}[x, H_1, \cdots, H_m, \overline{H}_1, \cdots, \overline{H}_m] / I}$$

$$I := \left\langle H_i + \overline{H}_i - x, \prod_{H \in \mathcal{H}} H \;\middle|\; \mathcal{H} \text{ is a set of some hyper facets s.t. } \cap \mathcal{H} = \phi \right\rangle$$

$H_i, \overline{H}_i$: hyper facet (generators)

$\cap \mathcal{H} = \phi$ (relations)