INTERNATIONAL CONFERENCE ON TORIC TOPOLOGY

Osaka City University, 29 May–3 June 2006

PROGRAMME AND ABSTRACTS OF TALKS

PROGRAMME OF TALKS

Monday, 29 May

09:00–09:30	Registration, Coffee, Opening remarks.
09:30–10:30	Robert MACPHERSON (Inst for Advanced Study) *Intersection homology and torus actions*
10:30–11:00	Coffee
11:00–11:45	Hiroshi KONNO (University of Tokyo) *Toric hyperKähler varieties*
11:45–12:00	Coffee
12:00–12:45	Tom BRADEN (University of Massachusetts) *Equivariant intersection cohomology of toric varieties and applications*
12:45–15:00	Lunch
15:00–15:30	Left side: Shintaro KUROKI (Osaka City University) *Hypertorus graph and its equivariant cohomology* Right side: Alexander GAIFULLIN (Moscow State University) *Local formulae for the Pontrjagin classes of combinatorial manifolds*
15:30–15:45	Coffee
15:45–16:15	Left side: Svjetlana TERZIĆ (University of Montenegro) *On invariant almost complex and complex structures on generalized symmetric spaces* Right side: Dong Youp SUH (KAIST) *Classification of quasi-toric manifolds and small covers over products of simplices*
16:15–16:45	Coffee
16:45–17:15	Left side: Rebecca GOLDIN (George Mason University) *The orbifold cohomology of hypertoric varieties* Right side: Jelena GRBIĆ (University of Aberdeen) *The homotopy type of the complement of a coordinate subspace arrangement*

Tuesday, 30 May

09:00–09:30	Coffee
9:30–10:30	Akio HATTORI (University of Tokyo) *Rigidity and invariance property of orbifold elliptic genus*
10:30–11:00	Coffee
11:00–11:45	Nicholas PROUDFOOT (University of Texas) *All the toric varieties at once*
11:45–12:00	Coffee
12:00–12:45	Tara HOLM (Univ of Connecticut/Cornell Univ) *Toric varieties and orbifolds in the symplectic category*
12:45–15:00	Lunch
15:00–15:30	Left side: Ivan ARZHANTSEV (Moscow State University) *Almost homogeneous toric varieties* Right side: Yasuhiko KAMIYAMA (Ryukyu University) *The symplectic volume of spatial polygon spaces*
15:30–15:45	Coffee
15:45–16:15	Left side: Dmitri TIMASHEV (Moscow State University) *Projective compactifications of reductive groups* Right side: Takahiko YOSHIDA (Tokyo University) *Twisted toric structure*
16:15–16:45	Coffee
16:45–17:15	Left side: Sam PAYNE (University of Michigan) *Toric vector bundles and branched covers of fans* Right side: Matthias FRANZ *Is every toric variety an M-variety?*
17:15–17:30	Coffee
17:30–18:00	Left side: Michael ENTOV (Technion, Haifa) *Intersection rigidity of fibers of moment maps of Hamiltonian torus actions* Right side: Mark HAMILTON (University of Calgary) *Quantization of toric manifolds via real polarizations*

Wednesday, 31 May

09:00–09:30	Coffee
09:30–10:30	Victor BUCHSTABER (Steklov Inst/U of Manchester) *Analogous polytopes, toric manifolds and complex cobordisms*
10:30–11:00	Coffee
11:00–12:00	Leonid POLTEROVICH (Tel Aviv University) *Quasi-states, Lagrangian fibrations and symplectic intersections*
12:00–12:15	Coffee
12:15–13:00	Dietrich NOTBOHM (University of Leicester) *Homology decompositions and Stanley-Reisner algebras*

Thursday, 1 June

09:00–09:30	Coffee
9:30–10:30	Askold KHOVANSKII (Univ of Toronto/IU Moscow) *A functional Log in the space of n-dimensional cycles and Parshin-Kato symbols*
10:30–11:00	Coffee
11:00–11:45	Paul BIRAN (Tel Aviv University) *Circle actions on algebraic varieties and projective duality*
11:45–12:00	Coffee
12:00–12:45	Christopher ALLDAY (University of Hawaii) *Cohomological aspects of symplectic Lie group actions*
12:45–15:00	Lunch
15:00–15:30	Left side: Margaret SYMINGTON (Georgia Tech/Mercer Univ) *Toric structures on near-symplectic manifolds* Right side: Takeshi KAJIWARA (Tohoku University) *Toric geometry and tropical geometry*
15:30–15:45	Coffee
15:45–16:15	Left side: Catalin ZARA (Univ of Massachusetts at Boston) *Hamiltonian GKM spaces and their moment graphs* Right side: Hui LI (University of Luxembourg) *The fundamental group of symplectic manifolds with hamiltonian Lie group actions*
16:15–16:45	Coffee
16:45–17:15	Left side: Jonathan WEITSMAN (U of California, Santa Cruz) *The Ehrhart formula for symbols* Right side: Jean-Claude HAUSMANN (University of Geneva) *Equivariant bundles over split* Γ*-space*
17:15–17:30	Coffee
17:30–18:00	Left side: Milena HERING (University of Michigan) *Syzygies of toric varieties* Right side: Takashi KIMURA (Boston University) *Stringy cohomology and stringy K-theory of symplectic orbifolds*

Friday, 2 June

09:00–09:30	Coffee
9:30–10:30	Nigel RAY (University of Manchester) *Toric topology and its categorical aspects*
10:30–11:00	Coffee
11:00–11:45	Susan TOLMAN (University of Illinois) *Graphs and equivariant cohomology: A (very) generalized Schubert calculus*
11:45–12:00	Coffee
12:00–12:45	Julianna TYMOCZKO (University of Michigan) *Permutation actions on equivariant cohomology*
12:45–15:00	Lunch
15:00–15:30	Left side: Parameswaran SANKARAN (Inst of Math Sci, India) *K-theory of quasi-toric manifolds* Right side: Linda CHEN (Ohio State University) *Towards the ample cone of moduli spaces*
15:30–15:45	Coffee
15:45–16:15	Left side: Tony BAHRI (Rider University) *On the stable spitting of complex coordinate subspace arrangements and related topics* Right side: Zhi LU (Fudan University) *Topology on graphs*
16:15–16:45	Coffee
16:45–17:15	Left side: David ALLEN (Iona College) *On the homotopy groups of toric spaces with applications to the homotopy type of* $\mathcal{Z}_K$. Right side: Alastair CRAW (Stony Brook University) *Fiber fans and toric quotients*

ABSTRACTS

Christopher ALLDAY (University of Hawaii)

Cohomological Aspects of Symplectic Lie Group Actions

Abstract. Let G be a compact connected Lie group acting symplecticly on a closed symplectic manifold (M, ω). Many interesting results concerning such actions can be proved quite easily using the cohomological methods pioneered by P. A. Smith, A. Borel and many others. For example, if there is a fixed point, and if M satisfies the weak Lefschetz condition, then the action is Hamiltonian. And, if the action is Hamiltonian (or, more weakly, c–Hamiltonian and uniform), and G is a torus of dimension r, then the fixed point set must have at least $r+1$ connected components. In this talk, I shall discuss several results of this kind, and, in some cases, indicate briefly how they are proven.
There are also results which appear to be cohomological, but which cannot be obtained by purely cohomological methods. The first of these was found by Frankel; and I shall discuss his and other such results from the cohomological point of view. I shall conclude by mentioning some open problems.

David ALLEN (Iona College)

On the homotopy groups of toric spaces with applications to the homotopy type of $\mathcal{Z}_K$.

Abstract. Given an n-dimensional, q-neighborly simple convex polytope P one has the associated Borel space, moment angle complex and family of toric manifolds that sit over P. Recently there has been much focus on the homotopy groups and homotopy type of the moment angle complex. Buchstaber and Panov determined the first non-trivial homotopy group of the Borel space using a particular cellular structure. We introduce the notion of relations among relations which allows for the determination of $R^1PBP_*(BTP)$ through a range. The stable and unstable co-action on $R^1PBP_*(BTP)$ is computed and shown to coincide with the co-action on a product of spheres whose dimensions depend on the combinatorics of P. As a result, the higher homotopy groups the Borel space can be determined through a range that was previously unknown. As an application, the homotopy type of a family of moment angle complexes (complex coordinate subspace arrangements complements) is determined and shown to be a wedge of spheres.

Ivan ARZHANTSEV (Moscow State University)

Almost homogeneous toric varieties

Abstract. The aim of the talk is to give an effective description of toric varieties X with an action of a semisimple group G such that there is an open orbit Gx in X and the complement $X \setminus Gx$ does not contain divisors. The talk is based on the joint work with J. Hausen, On embeddings of homogeneous spaces with small boundary (math.AG/0507557, to appear in J. Algebra). Here we develop the language of Cox rings originated from Cox's construction in toric geometry and extended to a class of normal varieties with a free finitely generated divisor class group in (F. Berchtold, J. Hausen, Cox rings and combinatorics, math.AG/0311115, to appear in Trans. AMS). This allows us to describe open equivariant embeddings of G/H into a normal G-variety X, where G is a simply connected linear algebraic group with only trivial character, H is a closed subgroup of G satisfying some mild restrictions, and the boundary $X \setminus (G/H)$ does not contain divisors.
In toric geometry, Cox's construction associates with any toric variety X an open subset U in a finite-dimensional vector space V such that X is obtained as the quotient of U by some torus. Under our assumptions, V is a prehomogeneous G-module. Note that the classification of such modules for a simple G is well known, but for a semisimple G the problem is much more difficult, and classifications are known only under some additional restrictions.
Having a prehomogeneous G-module V, we provide combinatorial data that parametrize 2-complete toric G-varieties X corresponding to V. Here we use combinatorial methods of Geometric Invariant Theory (GIT) and the language of bunches (F. Berchtold, J. Hausen, Bunches of cones in the divisor class group – a new combinatorial language for toric varieties, Inter. Math. Research Notices 6 (2004), 261-302), that makes this approach very effective.

Tony BAHRI (Rider University)

On the stable spitting of complex coordinate subspace arrangements and related topics

Abstract. A report of work in progress joint with Martin Bendersky, Fred Cohen and Sam Gitler. We investigate a splitting of the suspension of a moment angle complex into pieces related directly to the underlying simplicial complex. Included also will be a discussion of the topology of certain lens spaces from the moment angle complex point of view. The later relates to work in progress with Nigel Ray on the toric geometry of singular spaces.

Paul BIRAN (Tel Aviv University)

Circle actions on algebraic varieties and projective duality

Abstract. Let X be a smooth projective variety and Σ a hyperplane section of X. This talk will be concerned with relations between the symplectic topology of Σ and algebraic-geometric properties of the associated projective embedding of X in $\mathbb{C}P^N$. In particular we shall show that under the presence of a (holomorphic) circle action on Σ, in almost all cases the dual variety of X must be degenerate. Our approach uses techniques from symplectic topology.

Tom BRADEN (University of Massachusetts)

Equivariant intersection cohomology of toric varieties and applications

Abstract. The geometry of toric varieties is completely determined by the action of the defining torus, so it is natural that toric varieties give the most important test case for many programs to study the varieties with actions of tori. For instance, the theory of combinatorial intersection cohomology of fans developed by Barthel–Brasselet–Fieseler–Kaup, Bressler–Lunts, and Karu has shown that (equivariant) intersection cohomology sheaves on a toric variety X can be simply and effectively described in terms of the linear algebra of the fan, and in fact their theory works just as well for non-rational fans, for which the variety does not exist. In fact, using joint work with V. Lunts, this theory can be used to describe the category of all "mixed" sheaves on X. I will discuss these results, with a view toward how they might be extended to more complicated varieties such as flag varieties.

Victor BUCHSTABER (Steklov Institute/University of Manchester)

Analogous polytopes, toric manifolds and complex cobordisms

Abstract. The talk describe results of joint papers with Taras Panov and Nigel Ray. Our aim is to bring geometric and combinatorial methods to bear on the study omnioriented toric manifolds M, in the context of stably complex manifolds with compatible torus action. We interpret M in terms of combinatorial data (P, Λ), where P is the combinatorial type of an oriented simple polytope, and Λ is an integral matrix whose properties are controlled by P. In particular, we seek combinatorial criteria for comparing M with the class of non-singular projective toric varieties. We incorporate the theories of analogous polytopes, circle actions, formal group laws, and Hirzebruch genera. By way of application, we study conditions on (P, Λ) such that the corresponding toric manifold admits special unitary and level-N structures, and develop combinatorial formulae for the evaluation of genera in terms of subcircles of the torus action. We provide a discussion of the complex cobordism ring in the term of omnioriented toric manifolds

Linda CHEN (Ohio State University)

Towards the ample cone of moduli spaces

Abstract. I will give an overview of progress towards understanding the birational geometry of certain spaces, in particular their ample and nef cones of divisors and effective cones of curves. There is a (conjectured) combinatorial description of these cones on the moduli spaces of curves and the moduli spaces of stable maps which involve a natural torus action.

Alastair CRAW (Stony Brook University)

Fiber fans and toric quotients

Abstract. The chamber decomposition arising from the action of a subtorus on a quasiprojective toric variety gives rise to a polyhedral complex, and the fan obtained as the cone over the complex defines a toric variety. Which toric variety is it? And is it related to the original toric variety? I will describe joint work with Diane Maclagan that answers these questions by introducing the notion of the fiber fan and the toric Chow quotient.

Michael ENTOV (Technion, Haifa)

Intersection rigidity of fibers of moment maps of Hamiltonian torus actions

Abstract. There is a deep rigidity phenomenon in symplectic topology: certain subsets of a symplectic manifolds cannot be completely displaced from themselves by a Hamiltonian isotopy while it is possible to do so by just a smooth isotopy.
I will discuss how various interesting structures on symplectic manifolds coming from the Hamiltonian Floer homology allow to prove such a non-displaceability phenomenon for fibers of moment maps of Hamiltonian torus actions.
The talk is based on joint works with P. Biran and L. Polterovich.

Mattias FRANZ

Is every toric variety an M-variety?

Abstract. (joint work with Frédéric Bihan, Clint McCrory and Joost van Hamel, ArXiv `math.AG/0510228`)

Any (not necessarily smooth or compact) toric variety $X_\Sigma = X_\Sigma(\mathbb{C})$ comes with an involution τ, namely complex conjugation. Its fixed point set X^τ is the associated real toric variety $X_\Sigma(\mathbb{R})$. By the classical Smith–Thom inequality, the sum of the Betti numbers of $X_\Sigma(\mathbb{R})$ with coefficients in $\mathbb{F}_2$ cannot exceed the corresponding sum for $X_\Sigma(\mathbb{C})$,

$$\sum_k \dim H_k(X_\Sigma(\mathbb{R});\mathbb{F}_2) \leq \sum_k \dim H_k(X_\Sigma(\mathbb{C});\mathbb{F}_2).$$

Whenever equality holds in such a situation, the complex variety is called *maximal* or an *M-variety*.
I will present the conjecture that every toric variety is maximal with respect to homology with closed supports (also known as Borel–Moore homology). This is well-known if X_Σ is smooth and compact. Other cases in which we know our conjecture to hold are:

- equivariantly formal smooth toric varieties (this generalises the smooth compact case),
- compact (or equivariantly formal) toric varieties with isolated singularities, and
- toric varieties of dimension not greater than 3.

Moreover, the conjecture is supported by numerous examples.

Alexander GAIFULLIN (Moscow State University)

Local formulae for the Pontrjagin classes of combinatorial manifolds

Abstract. We present new local combinatorial formulae for polynomials in the Pontrjagin classes of combinatorial manifolds. We solve the following problem. Given a combinatorial manifold construct explicitly a rational simplicial cycle whose homology class is the Poincaré dual of a given polynomial in the Pontrjagin classes of the manifold. The coefficient at each simplex in the cycle obtained is determined solely by the combinatorial structure of the link of the simplex and can be computed from the given link by a finite algorithm.
Until recently there were two main combinatorial formulae for rational Pontrjagin classes. These are the Gelfand-MacPherson formula and the Cheeger formula. The Gelfand-MacPherson formula is the best known formula for a smooth manifold with a smooth triangulation. It is also very good in the category of CD-manifolds. Nevertheless this formula cannot be applied to an arbitrary combinatorial manifold. The Cheeger formula can be applied to an arbitrary combinatorial manifold. However the calculation by this formula includes computation of the specrum of a differential operator. Hence the cycle obtained cannot be computed by a finite algorithm. Also it is unknown whether this cycle is rational. The formula we present is the first formula that can be

applied to an arbitrary combinatorial manifold, gives rational cycles, and is algorithmically computable.
The approach we use is based on the Rokhlin-Shwartz-Thom definition of the rational Pontrjagin classes of a piecewise-linear manifold and on the concept of a *universal local formula*. For the first Pontrjagin class we use another method that gives a simpler formula. This method is based on the theory of bistellar moves.

Rebecca GOLDIN (George Mason University)

The orbifold cohomology of hypertoric varieties

Abstract. The geometry of hypertoric varieties is encoded in a hyperplane arrangement and some associated data. Under certain technical conditions, the hypertoric variety has at worst orbifold singularities. We will give a brief account of how to read off such singularities in a hypertoric orbifold, and how to compute the Chen-Ruan (orbifold) cohomology of these varieties. Our description involves looking at subhyperplane arrangements that correspond to the fixed points of isotropy groups, and is related to the work of Konno, Hausel-Sturmfels, and Goldin-Holm-Knutson. We will illustrate with pictures. Joint work with M. Harada.

Jelena GRBIĆ (University of Aberdeen)

The homotopy type of the complement of a coordinate subspace arrangement

Abstract. An arrangement $CA = \{L_1, \ldots, L_r\}$ in $\mathbb{C}^n$ is called coordinate if every L_i for $i = 1, \ldots, r$ is a coordinate subspace. We describe the unstable homotopy type of the complement $U(CA) := \mathbb{C}^m \setminus \cup_{i=1}^r L_i$ of a given coordinate subspace arrangement CA by combining the methods of classical homotopy theory and the new achievements of Toric Topology. As a corollary we obtain a new proof of the Golod result considering the rationality of the Poincaré' series of certain local rings.

Mark HAMILTON (University of Calgary)

Quantization of toric manifolds via real polarizations

Abstract. When geometric quantization using a real polarization is applied to a "nice enough" manifold, a result of Sniatycki says that the quantization can be found by counting certain objects, called Bohr-Sommerfeld fibres. Subsequently, several authors have taken this as motivation for counting Bohr-Sommerfeld fibres when studying the quantization of manifolds that are less "nice". In this talk, we discuss quantization of toric manifolds using a real polarization. In this

case Sniatycki's theorem does not apply, but results can be computed directly, and they are not exactly what we might expect. In particular, the quantization thus obtained is different from the quantization obtained using a Kaehler polarization.

Akio HATTORI (University of Tokyo)

Rigidity and invariance property of orbifold elliptic genus

Abstract. In recent years orbifold elliptic genus turned out to be a useful invariant for $\mathbb{Q}$-Gorenstein algebraic varieties in algebraic geometry. For instance if M/G is a global quotient of a complex manifold M by a finite group G admitting a crepant resolution of singularities its orbifold elliptic genus was conjectured to coincide with the elliptic genus of any crepant resolution of M/G. Restricted to orbifold Euler number or stringy Euler number, a special value of orbifold elliptic genus, the number is equal to the ordinary Euler number of a crepant resolution as was proved by Dixon, Harvey, Vafa and Witten. The fact is related to an observation of Mckay concerning the relation between minimal resolutions of quotient singularities $\mathbb{C}^2/G$ and the representations of G. The conjecture is now proved by Borisov-Libgober in more general contexts.

On the other hand it is known that orbifold elliptic genus are endowed with certain rigidity properties with respect to torus actions. In this talk I concentrate on torus orbifolds, topological analogues of toric varieties, and reveal the relation between rigidity and invariance property with respect to crepant morphisms for orbifold elliptic genus on torus orbifolds and toric varieties.

Algebro-geometric properties of a toric variety are translated to those of the fan associated to the variety. Topological counterparts of fans are multi-fans. On a simplicial multi-fan $\mathbb{Q}$-divisors are defined to be elements of degree two in its Stanley-Reisner ring. To every triple $(\Delta, \mathcal{V}, \xi)$ of a complete multi-fan Δ, a set of generating integral vectors $\mathcal{V}$ for each 1-dimensional cone, and a $\mathbb{Q}$-divisor ξ, the (equivariant) orbifold elliptic genus $\hat{\varphi}_{st}(\Delta, \mathcal{V}, \xi)$ is defined. Rigidity theorem says that, if ξ is a principal divisor, the orbifold elliptic genus vanishes. It is used to prove the invariance theorem which says that, if there is a crepant morphism $(\Delta', \mathcal{V}', \xi') \to (\Delta, \mathcal{V}, \xi)$, then $\hat{\varphi}_{st}(\Delta', \mathcal{V}', \xi')$ equals $\hat{\varphi}_{st}(\Delta, \mathcal{V}, \xi)$. In proving the invariance theorem, additivity property of orbifold elliptic genus that comes from a generalization of a character formula due to Borisov-Libgober is used to localize the problem.

Jean-Claude HAUSMANN (University of Geneva)

Equivariant bundles over split Γ-space

Abstract. For Γ and G compact Lie groups, we present, using the method of isotropy representations, a classification of Γ-equivariant principal G-bundles over a space X with a "split" Γ-action (i.e. an action with a section of $X \to X/\Gamma$). This includes equivariant bundle over toric manifolds (where Γ is a torus). The classification is best computable when G abelian but other cases are also interesting, like G of rank one. (Joint work with Ian Hambleton.)

Milena HERING (University of Michigan)

Syzygies of toric varieties

Abstract. Studying the equations defining the embedding of a projective variety and the higher relations (syzygies) between them is a classical problem in algebraic geometry. We give criteria for ample line bundles on toric varieties to give rise to a projectively normal embedding whose ideal is generated by quadratic equations and whose first q syzygies are linear. I will illustrate the interactions with the combinatorics of lattice polytopes and commutative algebra. Much of this has also appeared in a preprint with H. Schenck and G. Smith.

Tara HOLM (Univ of Connecticut/Cornell University)

Toric varieties and orbifolds in the symplectic category

Abstract. In the symplectic category, toric varieties are constructed as symplectic reductions of $\mathbb{C}^n$, using combinatorial data from their moment polytope. I will review the details of this construction, and discuss the cohomological techniques from symplectic geometry that allow us to understand the topology of these quotients. As time permits, I will introduce symplectic toric orbifolds and indicate how to compute their orbifold and Chen-Ruan orbifold cohomology.

Takeshi KAJIWARA (Tohoku University)

Toric geometry and tropical geometry

Abstract. We present a theory of tropical toric varieties, and, as applications, show an intersection theory on tropical projective toric surfaces, the tropical Bezout theorem, and so on.

YASUHIKO KAMIYAMA (RYUKYU UNIVERSITY)

The symplectic volume of spatial polygon spaces

ABSTRACT. Let M_n be the moduli space of polygons in $\mathbb{R}^3$ with n edges of length 1, modulo rotation. I will talk on the symplectic volume of M_n. The lines of calculations are as follows: An open dense subspace of M_n admits a T^{n-3}-action which preserves the symplectic form on M_n. We describe the moment map. Then using the Duistermaat-Heckman theorem, it will suffice to calculate the volume of a polytope.

ASKOLD KHOVANSKII (UNIVERSITY OF TORONTO/ INDEPENDENT UNIVERSITY OF MOSCOW)

A functional *Log* in the space of n-dimensional cycles and Parshin-Kato symbols

ABSTRACT. Let $f_1, \dots, f_{n+1}$ be $(n+1)$ meromorphic functions on an n-dimensional complex analytic space X. Denote by U the complement in X of the divisors of $f_1, \dots, f_{n+1}$ and the singular locus of X. Let $F = (C_0 \subset C_1 \subset \cdots \subset C_{n-1})$ be a flag of the irreducible subspaces of X with $\dim C_k = k$. Brilinski and McLaughlin defined [1] a symbol $\{f_1, \dots, f_{n+1}\}_F \in \mathbb{C}^*$ of the functions $f_1, \dots, f_{n+1}$ at the flag F. The symbol depends on components in the $(n+1)$-tuple $f_1, \dots, f_{n+1}$ in the following way : 1) it is multiplicative in each component, 2) it changes its value to the inverse element in $\mathbb{C}^*$ under an odd transposition of the components. The symbol is obtained by pairing of a certain cohomology class $(f_1, \dots, f_{n+1}) \in H^k(U, \mathbb{C}^*)$ and flag-localized homology class $\gamma_F \in H_k(U, \mathbb{Z})$. In the algebraic case over the field $\mathbb{C}$ the Brilinski–McLaughrin symbols coincide with the Parshin–Kato symbols and the reciprocity laws turn out to be direct corollaries from the properties mentioned above [1]. The construction of these symbols heavily uses analyticity of functions f_i and the space X.
I came across the Parshin–Kato symbols for the first time in a very surprising way. They appeared in the answer for a computation of the product of the roots in $(\mathbb{C}^*)^n$ of a system of n polynomial equations with generic Newton polyhedra [2]. I spend lots of time trying to understand that appearance of the symbols and trying to generalize and simplify their definition. I constructed a functional *Log* in the space of n dimensional cycles on a smooth real manifold U which depends on a smooth map $(f_1, \dots, f_{n+1}) : U \to (\mathbb{C}^*)^n$. The functional *Log* is a kind of a multidimensional version of the logarithmic function. As a function in $(f_1, \dots, f_{n+1})$ it satisfies all the symbol properties mentioned above. As a function on cycles it is an element of $H^k(U, \mathbb{C}^*)$ if and only if the $(n+1)$-form $\omega = df_1 \wedge \cdots \wedge df_{n+1}$ vanishes on U. In the case where U is a complex analytic n-dimensional manifold and $f_1, \dots, f_{n+1}$ are analytic functions this condition obviously holds. In that case the

Log functional coincides with the Brilinski–McLaughrin cohomology class $(f_1, \ldots, f_{n+1})$. Now I am trying to find a toric interpretation for the multidimensional Parshin–Kato symbols, analogous to my toric interpretation for the one-dimensional Weil symbols [3].

References

[1] Brylinski J.-L., McLaughlin D. A. *Multidimensional reciprocity laws*, J. reine angew. Math. 481 (1996), 125–147.

[2] Khovanskii A. *Newton polyhedrons, a new formula for mixed volume, productof roots of a system of equations*, in: Proceed. of a Conf. in Honour of V. I. Arnold, Fields Inst. Comm., vol. 24, Amer. Math. Soc., USA, 1999, pp. 325–364.

[3] Khovanskii A. *Newton polytopes, curves on toric surfaces, and inversion of Weil's theorem*, Russian Math. Surveys 52:6 (1997), 1251–1279.

Takashi KIMURA (Boston University)

Stringy cohomology and stringy K-theory of symplectic orbifolds

Abstract. Associated to a symplectic manifold (or even a stable almost complex manifold) with a finite group action is a stringy cohomology ring due to Fantechi-Goettsche whose coinvariants yield the so-called Chen-Ruan orbifold cohomology of the quotient symplectic orbifold. These symplectic invariants were introduced in terms of the Gromov-Witten theory of orbifolds. We present an elementary new definition of these stringy cohomology rings which removes all references to Riemann surfaces and their moduli. We then introduce a stringy K-theory, a K-theoretic version of these rings. Finally, we introduce a ring isomorphism between the stringy K-theory and the stringy cohomology, called the stringy Chern character, which is a deformation of the ordinary Chern character. Taking coinvariants yields a ring isomorphism between the orbifold K-theory, a K-theoretic version of orbifold cohomology, and the orbifold cohomology of the quotient symplectic orbifold. We also generalize these results to orbifolds which need not arise as global quotients by a finite group. As a consequence, we prove that the twisted orbifold K-theory of the symmetric product of a projective surface with trivial first Chern class is isomorphic to the ordinary K-theory of its resolution of singularities, the Hilbert scheme of points on the surface.

Hiroshi KONNO (University of Tokyo)

Toric hyperKähler varieties

Abstract. Toric hyperKähler varieties are hyperKähler or quaternionic analogues of usual toric varieties. They are constructed as hyperKähler quotients of quaternionic vector spaces by tori. They have

not only analogous properties to usual toric varieties, but also many properties from hyperKähler or complex symplectic geometries. In this talk I will describe the geometry of toric hyperKähler varieties as an intersection of real and complex symplectic geometry.

Shintaro KUROKI (Osaka City University)

Hypertorus graph and its equivariant cohomology

Abstract. Motivated by a result of Goresky-Kottwitz-MacPherson, Guillemin-Zara introduced the notion of a GKM-graph $\mathcal{G}$ and defined its (equivariant) cohomology which we denote by $H^*_T(\mathcal{G})$. An important fact is that if $\mathcal{G}$ is associated with an equivariantly formal T-space M such as a toric manifold where T is a torus group, then $H^*_T(\mathcal{G})$ is isomorphic to the equivariant cohomology $H^*_T(M)$ of M.
Maeda-Masuda-Panov introduced the notion of a torus graph as a combinatorial counterpart of a torus manifold introduced by Hattori-Masuda. A torus graph is not necessarily a GKM-graph but the equivariant cohomology can be defined similarly to a GKM-graph. They proved that the equivariant cohomology of a torus graph is isomorphic to the face ring of a simplicial poset dual to the torus graph. It is also true that if a torus graph $\mathcal{G}$ is associated with an equivariantly formal torus manifold M, then $H^*_T(\mathcal{G})$ is isomorphic to $H^*_T(M)$.
In this talk we will introduce the notion of a *hypertorus graph* and show that its equivariant cohomology is isomorphic to an algebra defined by some combinatorial structure of the hypertorus graph. A labeled graph associated with a hypertoric manifold or a cotangent bundle over a torus manifold is an example of a hypertorus graph. As an application of our result, we prove a result about the equivariant cohomology of a GKM-graph which allows *legs*. Here leg means an out going half line from one vertex.

Hui LI (University of Luxembourg)

The fundamental group of symplectic manifolds with hamiltonian Lie group actions

Abstract. Let (M, ω) be a connected, compact symplectic manifold equipped with a Hamiltonian G action, where G is a connected compact Lie group. Let ϕ be the moment map. Then, as fundamental groups of topological spaces, $\pi_1(M) = \pi_1(M_{red})$, where M_{red} is the symplectic quotient at any value of the moment map ϕ.

Zhi LU (Fudan University)

Topology on graphs

Abstract. Motivated by the work of Goresky, Kottwitz and MacPherson, we find that any effective $(\mathbb{Z}_2)^k$-action on a closed manifold M^n fixing a finite set can induce a regular graph Γ_M with some properties (precisely, those properties are presented by a natural map α on Γ_M). Then we reformulate the map α and give an abstract definition for (Γ_M, α). Next, we give the notion of a face on (Γ, α). When $n = k$, all faces of (Γ, α) form a simplicial posets with respect to reversed inclusion, so that (Γ, α) has a geometric realization $|(\Gamma, \alpha)|$. In this talk, we mainly consider the following two basic problems: (I) under what condition, is the geometric realization $|(\Gamma, \alpha)|$ a closed manifold? (II) for any n-dimensional closed manifold N, is there some (Γ, α) such that N is homeomorphic to $|(\Gamma, \alpha)|$?

Robert MACPHERSON (Institute for Advanced Study)

Intersection Homology and Torus Actions

Abstract. If an algebraic torus acts on a complex algebraic variety, intersection homology provides an associated algebraic-combinatorial “sheaf” on the graph constructed from the zero and one dimensional orbits. This talk will discuss this sheaf, and its behavior under algebraic maps. This is joint work with Tom Braden.

Dietrich NOTBOHM (University of Leicester)

Homology decompositions and Stanley-Reisner algebras

Abstract. Stanley-Reisner algebras are the main invariant to study the combinatorics of polytopes or simplicial complexes. There are several ways to construct topological realizations; i.e. a topological space, whose cohomology is isomorphic to a given Stanley-Reisner algebra. We will present a further construction (homotopy colimits of nice diagrams) which allows to bring methods of homotopy theory into the play. Using methods of homotopy theory, we will construct a particular vector bundle over such realizations which is closely related to the tangent bundle of toric manifolds. As applications, we will characterize depth conditions on Stanley-Reisner algebras in terms of the combinatorics of the underlying simplicial complexes and relate questions about colourings of simplicial complexes to geometric properties of the vector bundle.
This is partly joint work with Nigel Ray.

Sam PAYNE (University of Michigan)

Toric vector bundles and branched covers of fans

Abstract. To each equivariant vector bundle on a toric variety, we associate a "branched cover" of the associated fan, together with a piecewise-linear function on it. This combinatorial approach leads to a simple description of the moduli stack of equivariant vector bundles with fixed equivariant Chern class on an arbitrary toric variety, and is useful for computations and for constructing examples. We use these branched covers to show that certain complete (singular, nonprojective) toric threefolds which were known to have no nontrivial line bundles also have no nontrivial equivariant vector bundles of rank less than four. It is not known whether these varieties have any nontrivial vector bundles at all.

Leonid POLTEROVICH (Tel Aviv University)

Quasi-states, Lagrangian fibrations and symplectic intersections

Abstract. We establish a link between symplectic topology and the theory of quasi-states – a recently emerged branch of functional analysis originated in quantum mechanics. In the symplectic context quasi-states can be viewed as an algebraic way of packaging certain information contained in Floer theory. We present applications to the study of (singular) Lagrangian fibrations and in particular to detecting those fibers which cannot be displaced by a Hamiltonian diffeomorphism. The talk is based on joint works `math.SG/0205247`, `math.SG/0410338` with M. Entov, `math.SG/0307011` with P. Biran and M. Entov, and a work with progress with M. Entov and F. Zapolsky.

Nicholas PROUDFOOT (University of Texas)

All the toric varieties at once

Abstract. Let V be a linear representation of an algebraic torus T. The various GIT quotients of V by T are toric varieties, and the holomorphic symplectic quotient of the cotangent bundle of V by T is a hypertoric variety. I will give one of many perspectives on the topology of hypertoric varieties, and explain why studying this space is like studying all of the toric varieties at once.

Nigel RAY (University of Manchester)

Toric topology and its categorical aspects

Abstract. I shall begin my talk by proposing a list of mathematical activities which should lie at the heart of any definition of toric

topology. I will build on original proposals made with Taras Panov (around 1999), and include further suggestions of Victor Buchstaber; the list includes topics which are algebraic, combinatorial, differential, geometric, and homotopy theoretic in nature. I shall try to make our ideas accessible to a general audience by relating them to a specific set of basic examples, including projective spaces and bounded flag manifolds. During the remainder of the talk, I shall use the same examples to explain why I believe that certain aspects of category theory should be added to the list. I shall argue that it plays two roles; one local, and one global.

The local role arises from the origins of the subject in algebraic geometry, where the orbit quotient of a toric variety is a simple convex polytope. The variety may be expressed in terms of combinatorial data associated to the boundary K of the dual simplicial polytope. The faces of K form the objects of a finite category $\mathrm{CAT}(K)$, which may then be used to construct many of the spaces of toric topology; these include the moment angle complexes $\mathcal{Z}_K$, toric manifolds M, and Davis-Januszkiewicz spaces $DJ(K)$. We denote such spaces generically by X_K, and refer to them as *toric spaces*. The constructions lead us naturally from the local to the global viewpoint, because they involve $\mathrm{CAT}(K)$-diagrams in the category of topological spaces. A toric space may then, for example, be the colimit or homotopy colimit of some such diagram.

When we apply algebraic invariants to solve topological problems we aim to interpret the procedure functorially, by mapping toric spaces into a suitable algebraic category ALG, where calculations may be more straightforward. In this language, the invariants of X_K are constructed from the corresponding $\mathrm{CAT}(K)$-diagrams in ALG, by forming colimits or homotopy colimits in the appropriate algebraic sense. A prime example is given by Sullivan's commutative cochain functor A_{PL}, for which ALG is the category of commutative differential graded algebras over the rationals $\mathbb{Q}$. This example has the additional structure of a *Quillen model category*, which allows us to manufacture homotopy colimits in a systematic, geometrically motivated fashion, using familiar algebraic concepts. Other such categories that I shall mention include differential graded coalgebras over a ring, and differential graded Lie algebras over $\mathbb{Q}$.

My overall aim is to explain how these abstract ideas actually reflect the underlying geometry. As well as outlining concrete calculations associated to the basic examples, I shall appeal to supporting evidence provided by various collaborations with Panov, Dietrich Notbohm, and Rainer Vogt.

Parameswaran SANKARAN (Inst of Math Sciences, India)

K-theory of quasi-toric manifolds

Abstract. In this talk I shall present the results of recent joint work with V. Uma in which we describe the complex K-ring of a quasi-toric manifold in terms of generators and relations. As an example, I shall describe the K-ring of Bott-Samelson varieties.

Dong Youp SUH (Korea Advanced Institute of Science and Technology)

Classification of quasi-toric manifolds and small covers over products of simplices

Abstract. A quasi-toric manifold (resp. small cover) over an n-dimensional polytope P is a smooth $2n$-dimensional real manifold (resp. n-dimensional manifold) M with an n-dimension torus $G = T^n = S1 \times \cdot \times S1$ action (resp. n-dimensional $\mathbb{Z}_2$ torus $G = \mathbb{Z}_2 \times \cdots \times \mathbb{Z}_2$ action), which is locally isomorphic to the standard representation with a projection map $\pi\colon M \to P$ such that the fiber of π are the G orbits. Hence the orbit space M/G is homeomorphic to P. In this talk we will give some classification results on quasi toric manifolds and small covers over P when $P = \Delta^{n_1} \times \cdots \times \Delta^{n_m}$ is a product of simplices. Indeed, we show that two quasi-toric manifolds M_1 and M_2 are G-diffeomorphic if and only if $H_G(M_1) \cong H_G(M_2)$ as $H^*(BG)$-algebra. A similar result also holds for small covers. Moreover we show that every small cover is G-diffeomorphic to an iterated real projective space bundle. An iterated complex(resp. real) projective space bundle is constructed from a complex (res. real) projective space and a complex (resp. real) vector bundle over it. Then consider the corresponding complex (resp real) projective space bundle, and another vector bundle over it. We then take the corresponding projective space bundle, and the resulting space after a finite iteration of such process is called an iterated complex (ressp. real) projective space bundle. The Bott tower is a special case of an iterated projective space bundle. We also give some nonequivariant classification results on quasi-toric (resp. small cover) manifolds.

Margaret SYMINGTON (Georgia Tech University/ Mercer University)

Toric structures on near-symplectic manifolds

Abstract. A near-symplectic manifold is a four-manifold equipped with a two-form that is symplectic on the complement of a union of circles and that vanishes “nicely” along the circles. In this talk I will discuss what closed four-manifolds admit a near-symplectic structure that is invariant under a torus action, or more generally, compatible with a Lagrangian fibration with toric singularities. In particular, toric near-symplectic manifolds are classified by generalized moment map images. The study of such structures on near-symplectic manifolds is motivated by Taubes' program to develop Gromov-Witten type invariants for near-symplectic manifolds and recent calculations of Gromov-Witten invariants of toric manifolds in terms of graphs in moment map images (due to Parker using symplectic field theory, and Mikhalkin using tropical algebraic geometry). This is joint work with David Gay.

Svjetlana TERZIĆ (University of Montenegro)

On invariant almost complex and complex structures on generalized symmetric spaces

Abstract. In their work from 1958 Borel and Hirzebruch provided a way to describe invariant almost complex and complex structures on compact homogeneous spaces. They showed that on these spaces invariant almost complex structures as well as their integrability can be described by looking at the roots of the corresponding group and subgroup. In the same paper, using the root theory, they also provided the way to compute characteristic classes, and in particular Chern classes, of compact homogeneous spaces.
In this talk we consider generalized symmetric spaces. Using some our earlier results on this spaces we show that the above theory can be used to obtain explicit description of their invariant almost complex and complex structures as well as for computation of their Chern classes.
This also makes us possible to put some more light on question of topological (non) invariance of Chern numbers.

Dmitri TIMASHEV (Moscow State University)

Projective compactifications of reductive groups

Abstract. We study equivariant projective compactifications of reductive groups obtained by closing the image of a group in the space of operators of a projective representation. The varieties obtained in this way are a direct generalization of projective toric varieties. Their geometry is controlled by the weight polyhedra of the respective representations just in the same way as the geometry of projective toric varieties is controlled by their Newton polyhedra. We describe the structure and the mutual position of their orbits under the action of the doubled group by left/right multiplications, the local structure in a neighborhood of a closed orbit, and obtain some conditions of normality and smoothness of a compactification. Our methods use the theory of equivariant embeddings of spherical homogeneous spaces and of reductive algebraic semigroups.

Susan TOLMAN (University of Illinois)

Graphs and equivariant cohomology: A (very) generalized Schubert calculus

Abstract. Let a torus act on a compact symplectic manifold M in a Hamiltonian fashion with isolated fixed points; assume that there exists an invariant Palais-Smale metric. (For example, let M be a flag variety). We associate a labelled graph to M, and show that the equivariant cohomology ring of M can be computed by (appropriately) counting paths in this graph. Joint work with R. Goldin.

Julianna TYMOCZKO (University of Michigan)

Permutation actions on equivariant cohomology

Abstract. We discuss a way to construct permutation actions on the equivariant cohomology of various kinds of subvarieties of the flag variety. To do this, we review the Goresky-Kottwitz-MacPherson (GKM) approach to equivariant cohomology, which describes the equivariant cohomology of a suitable variety in terms of the variety's moment graph (a combinatorial graph obtained from the moment map). The permutation action on the equivariant cohomology can be described directly in terms of (combinatorial) graph automorphisms. We describe the representations that can be constructed, as well as many open questions.

Jonathan WEITSMAN (Univ of California, Santa Cruz)

The Ehrhart formula for symbols

Abstract. We prove an analog of the Ehrhart formula for symbols of pseudodifferential operators, using the Euler-Maclaurin formula with remainder.

Takahiko YOSHIDA (Tokyo University)

Twisted toric structure

Abstract. It is shown by Delzant that there is a one-to-one correspondence between symplectic toric manifolds and Delzant polytopes, and through this correspondence, various researches on the relationship between symplectic geometry, theory of transformation groups, topology, and combinatorics has been carried out. Recently, some generalizations are also considered. In this talk, as one of such generalizations, we shall introduce the notion of twisted toric manifolds, and prove the classification theorem. If we have a time, we also give the method to compute the fundamental group and cohomology groups.

Catalin ZARA (University of Massachusetts at Boston)

Hamiltonian GKM spaces and their moment graphs

Abstract. This talk will describe some beautiful connections between Symplectic Geometry and Combinatorics. For Hamiltonian GKM spaces, a lot of geometric and topological information about the space is encoded in the moment graph, and I shall describe how one can recover this information from the combinatorics of the moment graph.