BRITISH-RUSSIAN SEMINAR
ON TORIC TOPOLOGY AND HOMOTOPY THEORY

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ABSTRACTS

Semyon Abramyan

Whitehead products and Hurewicz homomorphism for moment-angle complexes

Matthew Burfitt

Integral cohomology of the free loop space of complete flag manifolds.

A complete flag manifold is the quotient of a Lie group by its maximal torus and is one of the nicer examples of homogeneous spaces. Related objects are studied in different areas of mathematics and mathematical physics. In topology, the study of free loop spaces on manifolds has two folded motivation. First there is a relation between geometrically distinct periodic geodesics on a manifold and their free loop spaces, originally studied by Gromoll and Meyer in their 1969 paper. More recently the study of string topology, in particular the Chas–Sullivan loop product, has been an active area of research with connection to interesting areas in algebraic topology including topological quantum field theory, operads and topological cyclic homology. In this talk I will discuss my work on the cohomology of the free loop space of complete flag manifolds. I will explain my results in the case of the special unitary group $SU(n)$, as this is complicated enough case to illustrate the main ideas but at the same time technically the simplest one.

Nikolay Erokhovets

Toric topology, and combinatorics of fullerenes and Pogorelov polytopes

Xin Fu

Homotopy fibrations in toric topology

Let $f\colon K \to L$ be a simplicial map between finite simplicial complexes. Then f induces a continuous map $g\colon (S^1,*)^K \to (S^1,*)^L$ between polyhedral products using the multiplicative structure on S^1. This can be seen as a homotopy theoretical generalisation of the Bestvina–Brady construction.

In this talk, I will consider the homotopy fibre of the map g and in certain cases, describe its homotopy type.

Dmitry Gugnin

On integral cohomology rings of symmetric products

The talk is based on the author's arXiv preprint 1502.01862. The symmetric product of a Hausdorff space X is the quotient space $Sym^n X := X^n/S_n$. There are lots of papers by Dold, Thom, Nakaoka, Milgram, Macdonald and many other mathematicians devoted to cohomology of symmetric products. The fundamental fact is that the rational cohomology ring $H^*(Sym^n X;\mathbb{Q})$ of the symmetric product is a rather simple functor of the initial ring $H^*(X;\mathbb{Q})$ (for connected CW-complexes X of finite homology type).

The main result to be presented at the talk is the author's theorem, which states that the ring $H^*(Sym^n X;\mathbb{Z})/Tor$ is a functor of the ring $H^*(X;\mathbb{Z})/Tor$ (for connected CW-complexes X of finite homology type). Moreover, we will give an explicit description of this functor. The nontriviality of the fact is that the functor $H^*(-;\mathbb{Z})/Tor$ is more subtle than $H^*(-;Q)$: there exist manifolds with torsion free cohomology having isomorphic rational cohomology rings and nonisomorphic integral cohomology rings.

Abigail Linton

Massey products in toric topology

Massey products are secondary operations defined on differential graded algebras, such as on the cohomology rings of a topological space. Although difficult to describe, there are a number of applications of Massey

products in topology, geometry, algebra and combinatorics. For example, a triple Massey product detects the non-trivial linking in the Borromean rings, which cannot be detected by cup products in cohomology.
Toric topology is the study of spaces arising from m-torus actions on topological spaces, whose orbit spaces have a rich combinatorial structure. A key example of such spaces are moment-angle complexes $\mathcal{Z}_{\mathcal{K}}$, whose T^m-orbit space is a simplicial complex $\mathcal{K}$ on m vertices. In this talk, I detect Massey products on moment-angle complexes and describe them in combinatorial terms.

Ingrid Membrillo-Solis
Homotopy types of gauge groups related to certain 7-manifolds

Let X be a path-connected pointed topological space and let G be a topological group. Given a principal G-bundle over X, $P \to X$, the gauge group is the group of G-equivariant automorphisms of P that fix X. The study of the topology of gauge groups when X is a low dimensional manifold has played a prominent role in mathematics and mathematical physics over the last thirty years. In 2011, however, Donaldson and Segal established the mathematical set-up to construct gauge theories using principal G-bundles over high dimensional manifolds. In this talk I will present some results on the homotopy theory of gauge groups when X is a certain 2-connected 7-manifold and G is a simple compact simply connected Lie group.

Tse-Leung So
Homotopy types of gauge groups over 4-manifolds

Gauge groups originate from physics and they have many applications in mathematics, such as classification of 4 differentiable manifolds. Given a Lie group G, a gauge group is defined to be the group of G-equivariant automorphisms of a principal G-bundle fixing its base manifold. In general gauge groups are difficult to compute. In this talk, I will discuss the homotopy types of gauge groups over certain non-simply connected 4-manifolds.

Dmitry Ulyumdzhiev

On cohomology of small covers over polytopes

In this talk I am going to give a brief introduction to the topology of special class of manifolds which is called small covers over simple polytope. It was originally introduced by M. W. Davis and T. Januszkiewicz in pair with quasi-toric manifolds. After a short review of recent results on small covers, I hope to discuss its homological properties, which was also studied by Davis and Januszkiewicz. Unfortunately, their approach has some important inaccuracy, which I am going to discuss too.

Yakov Veryovkin

Pontryagin algebras of some moment-angle complexes

We consider the problem of describing the Pontryagin algebra (loop homology) of moment-angle complexes and manifolds. The moment-angle complex $\mathcal{Z}_K$ is a cell complex built of products of polydiscs and tori parametrised by simplices in a finite simplicial complex K. It has a natural torus action and plays an important role in toric topology. In the case when K is a triangulation of a sphere, $\mathcal{Z}_K$ is a topological manifold, which has interesting geometric structures. Generators of the Pontryagin algebra $H_*(\Omega\mathcal{Z}_K)$ when K is a flag complex have been described in the work of Grbić, Panov, Theriault and Wu. Describing relations is often a difficult problem, even when K has a few vertices. Here we describe these relations in the case when K is the boundary of a pentagon or a hexagon. In this case, it is known that $\mathcal{Z}_K$ is a connected sum of products of spheres with two spheres in each product. Therefore $H_*(\Omega\mathcal{Z}_K)$ is a one-relator algebra and we describe this one relation explicitly, therefore giving a new homotopy-theoretical proof of McGavran's result. An interesting feature of our relation is that it includes iterated Whitehead products which vanish under the Hurewicz homomorphism. Therefore, the form of this relation cannot be deduced solely from the result of McGavran.